Multi-Scale Computational Homogenization of Plain-Weave Textile Composites

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Overview

1. Motivation

2. Multi-length Scale Analysis

3. Computational Unit Cell Homogenization

4. Conjugate, Symmetric Stress-Strain Measures

5. Transversely-Isotropic Hyper-Elastic Model

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7. Conclusions and Ongoing Work

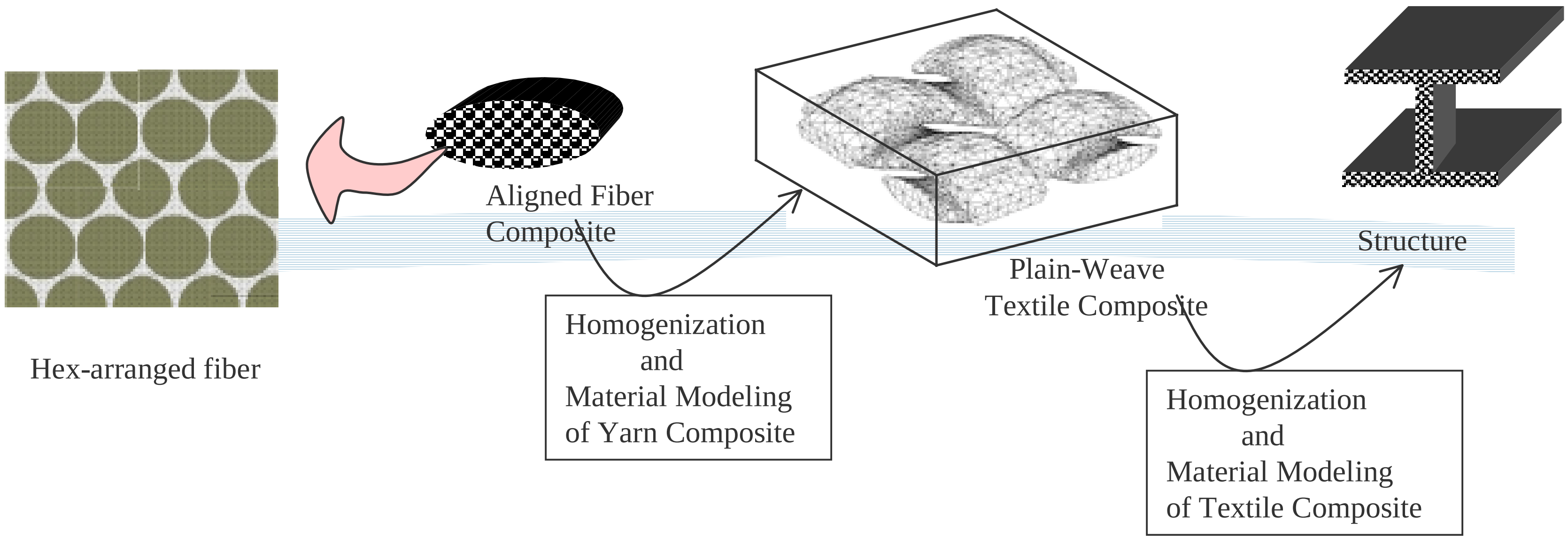
Motivation

• Develop unit-cell analysis as a tool to study property-structure characteristics of hierarchical textile composites

• Specific Challenges

1. Modeling/meshing of material structure
2. High cost of unit-cell analysis
3. Finite deformation homogenization (conjugacy)
4. Constitutive modeling (finding appropriate forms)

Multi-Scale Analysis Process



Material Scale Decompositions of Stress and Deformation

- Decomposition material/ macro stress/ deformation

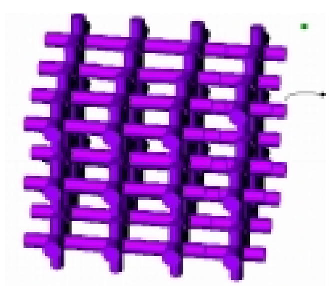
$$\boldsymbol{\tau}(\mathbf{X}) = \boldsymbol{\Sigma} + \boldsymbol{\tau}^*(\mathbf{X}); \qquad \mathbf{F}(\mathbf{X}) = \boldsymbol{\Phi} + \mathbf{F}^*(\mathbf{X});$$

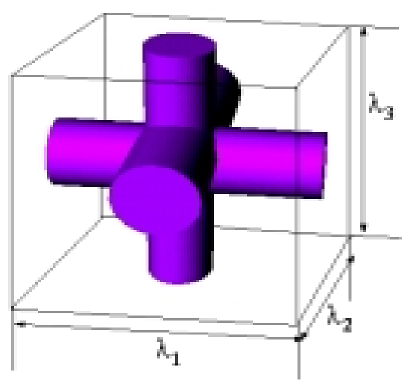
$$\boldsymbol{\Sigma} = \langle \boldsymbol{\tau} \rangle = \frac{1}{V}\int_{\Omega_s} \boldsymbol{\tau}\, d\Omega_s; \quad \boldsymbol{\Phi} = \langle \mathbf{F} \rangle = \frac{1}{V}\int_{\Omega_s} \mathbf{F}\, d\Omega_s;$$

$$\langle \boldsymbol{\tau}^* \rangle = \mathbf{0}; \qquad \langle \mathbf{F}^* \rangle = \mathbf{0}$$

- Periodicity of material scale stress and deformation fields:

$$\boldsymbol{\tau}(X_I) = \boldsymbol{\tau}(X_I + n\lambda_I); \quad \mathbf{F}(X_I) = \mathbf{F}(X_I + n\lambda_I); \quad (I = 1,2,3)$$





Computational Unit-Cell Homogenization

- Deformation controlled unit-cell analysis
  - Apply macro-scale deformation field $\mathbf{\Phi}$
    solve for periodic displacement field $\mathbf{u}^{*}_{\text{per}}(\mathbf{X})$;
    such that $\mathbf{u}(\mathbf{X}) = \mathbf{\Phi} \cdot \mathbf{X} + \mathbf{u}^{*}_{\text{per}}(\mathbf{X})$ satisfies stress equilibrium $\nabla \cdot \boldsymbol{\tau} = 0$

.

- Homogenized macro-scale stress / deformation

$$\mathbf{\Sigma} = \langle \boldsymbol{\tau} \rangle = \frac{1}{V} \int_{\Omega_s} \boldsymbol{\tau} \, d\Omega_s ; \quad \mathbf{\Phi} = \langle \mathbf{F} \rangle = \frac{1}{V} \int_{\Omega_s} \mathbf{F} \, d\Omega_s$$

- Determine homogenized material constitutive model out of macroscopic stress and deformation relations; $\mathbf{\Sigma} = \mathbf{\Sigma}(\mathbf{\Phi})$

Symmetric and Conjugate Stress, Strain Measures

- Conjugacy between averaged deformation gradient and averaged nominal stress was demonstrated by Nemat-Nasser (2000).

$$\left\langle \mathrm{P}_{\mathrm{Ji}} \dot{\mathrm{F}}_{\mathrm{iJ}} \right\rangle = \left\langle \mathrm{P}_{\mathrm{Ji}} \right\rangle \left\langle \dot{\mathrm{F}}_{\mathrm{iJ}} \right\rangle$$

- Modified symmetric stress and strain measures

$$\hat{\mathbf{S}} \equiv \langle \mathbf{P} \rangle \langle \mathbf{F} \rangle^{-T}$$ Macro 2nd P-K Stress Tensor;

$$\hat{\mathbf{E}} \equiv \tfrac{1}{2}\left(\langle \mathbf{F} \rangle^{T} \langle \mathbf{F} \rangle - \mathbf{I} \right)$$ Macro Green-Lagrange Strain Tensor

$\Rightarrow$ satisfy conjugacy in strain energy

$$\hat{\mathrm{S}}_{\mathrm{IJ}} \dot{\hat{\mathrm{E}}}_{\mathrm{IJ}} = \left\langle \mathrm{P}_{\mathrm{Ji}} \right\rangle \left\langle \dot{\mathrm{F}}_{\mathrm{iJ}} \right\rangle = \left\langle \mathrm{P}_{\mathrm{Ji}} \dot{\mathrm{F}}_{\mathrm{iJ}} \right\rangle = \left\langle \mathrm{S}_{\mathrm{IJ}} \dot{\mathrm{E}}_{\mathrm{IJ}} \right\rangle$$

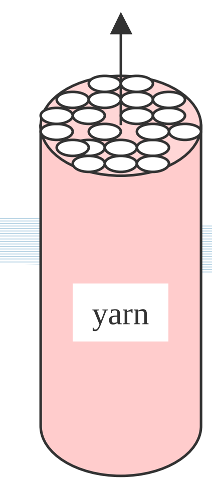
Transversely-Isotropic Hyperelastic Model

- Transversely-Isotropic Hyper-elastic model by Bonet and Burton (1998)

$$\psi = \psi(I_1, I_2, I_3, I_4, I_5) = \psi_{inh} + \psi_{ti}$$

$$\psi_{inh} = \tfrac{1}{2}\mu(I_1 - 3) - \mu \ln J + \tfrac{1}{2}\lambda(J-1)^2$$

$$\psi_{ti} = [\alpha + \beta \ln J + \gamma(I_4 - 1)](I_4 - 1) - \tfrac{1}{2}\alpha(I_5 - 1)$$



where

$$I_1 = \mathrm{tr}(\mathbf{C}); \quad I_2 = \mathbf{C} : \mathbf{C}; \quad I_3 = \det(\mathbf{C}) = J^2;$$

$$I_4 = \mathbf{A} \cdot \mathbf{C} \cdot \mathbf{A}; \quad I_5 = (\mathbf{A} \cdot \mathbf{C}) \cdot (\mathbf{C} \cdot \mathbf{A})$$

and **A** is material director of aligned fibers in material configuration

Estimation of Coefficients for Aligned Fiber Model

- Coefficients for Bonet and Burton model $\boldsymbol{\alpha} = (\lambda, \mu, \alpha, \beta, \gamma)$
- Deformation-controlled homogenization

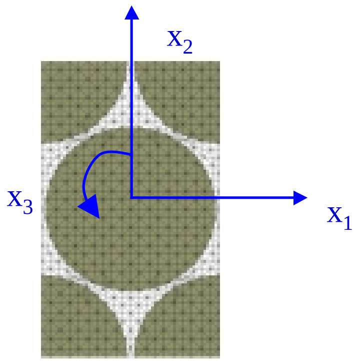
$$\min_{\boldsymbol{\alpha}} \sum_{k} \int_{t_1}^{t_2} \left\| \frac{\hat{S}^k(t) - \tilde{S}^k(\boldsymbol{\alpha}, t)}{\hat{S}^k(t)} \right\| dt$$

where

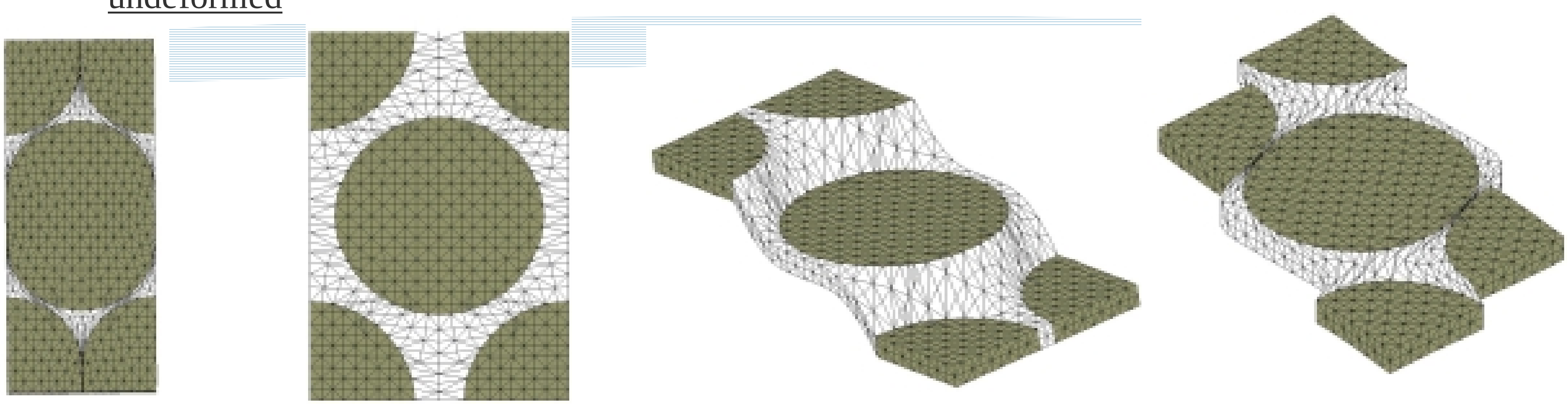
$\hat{S}^k(t)$ is homogenized stress under k_{th} strain-controlled case

$\tilde{S}^k(\boldsymbol{\alpha}, t)$ is 2nd PK stress from proposed model.

Unit-Cell of Hexagonally Packed Aligned Fibers

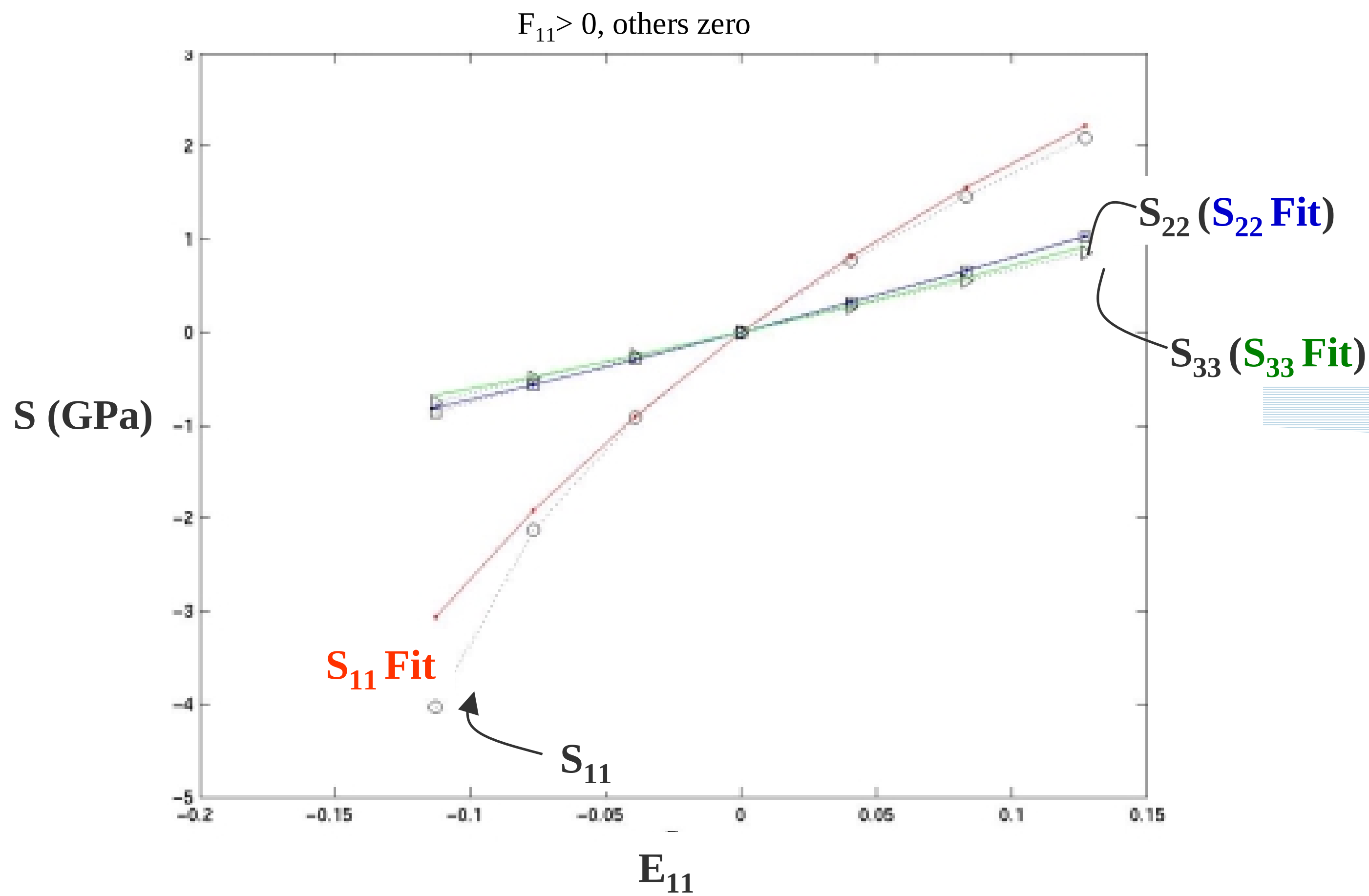


undeformed

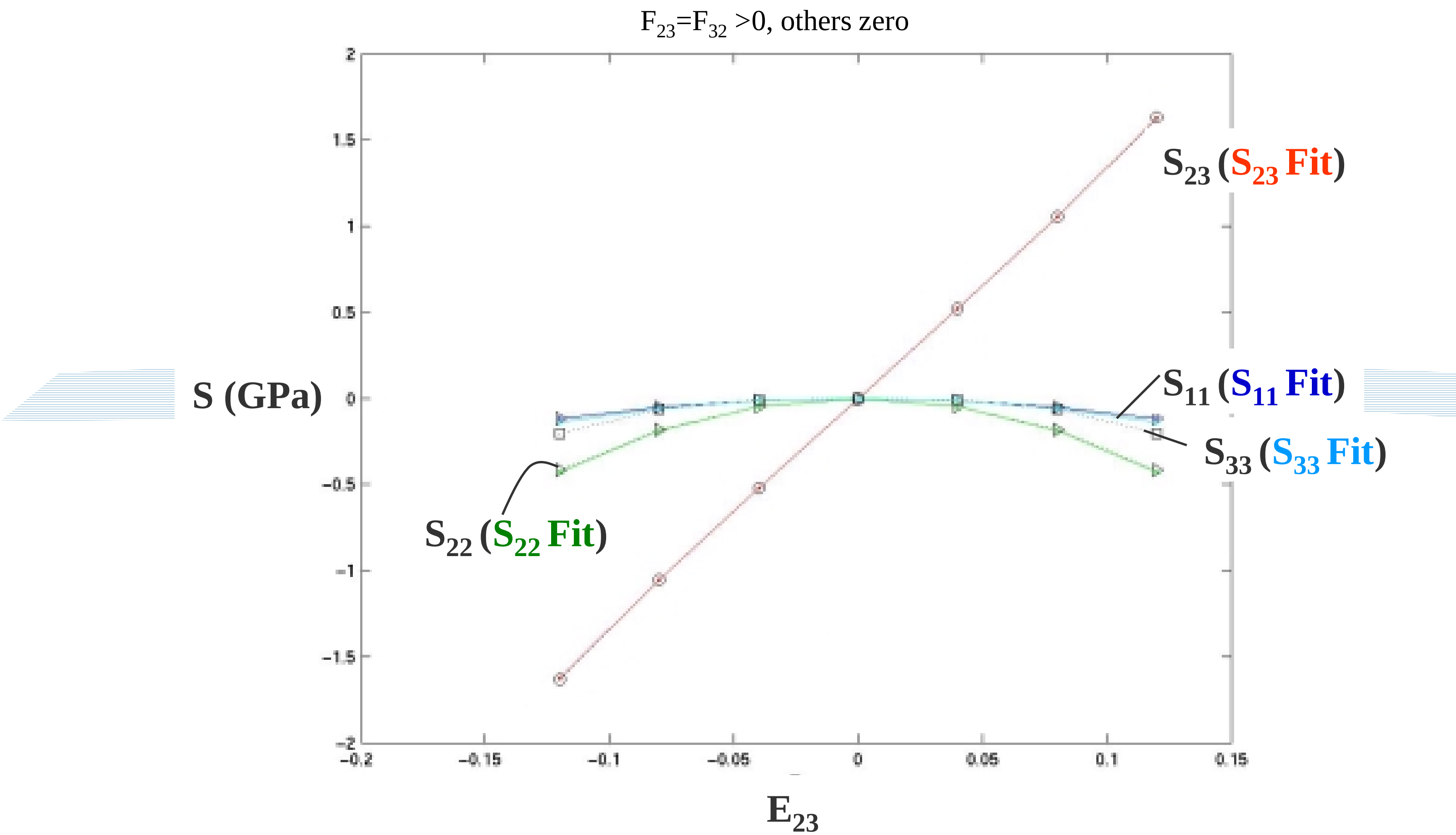


(a) In-plane compression (b) In-plane tension (c) Out-of-plane shear (d) In-plane shear

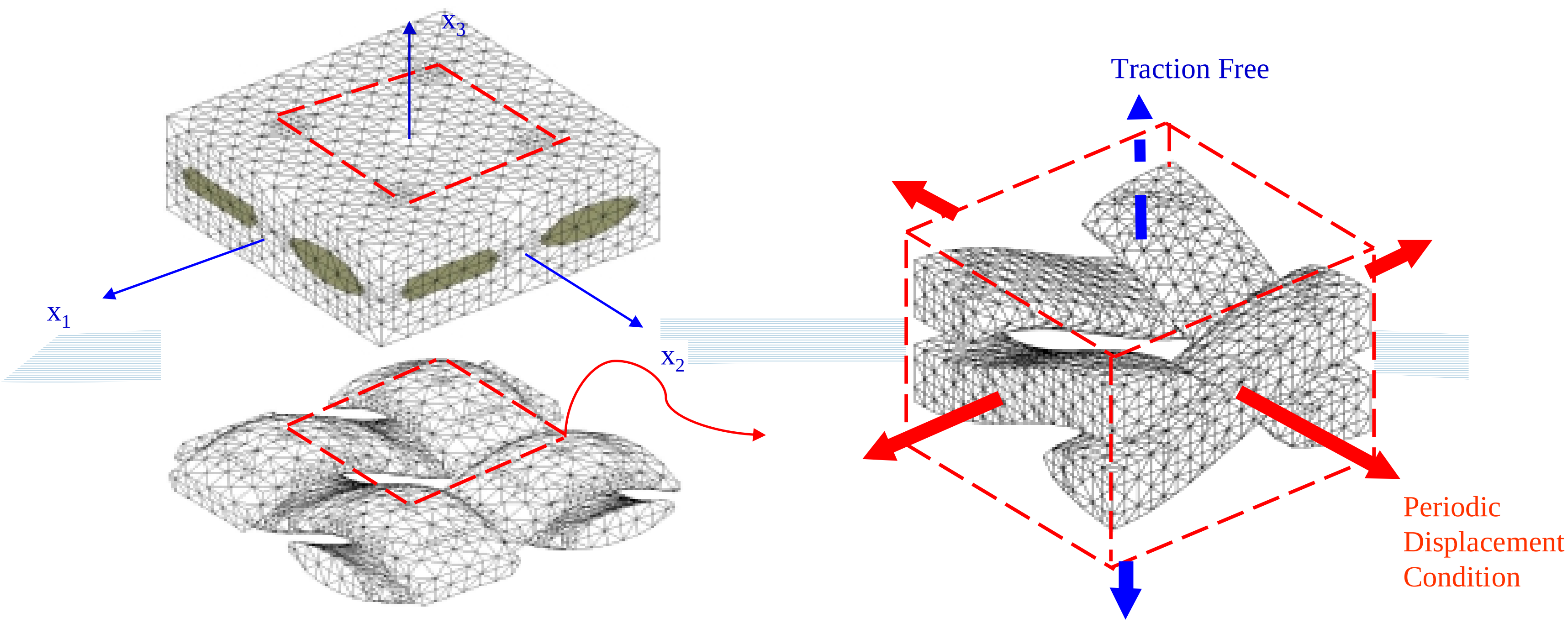
Stress-Strain Behavior of Aligned-Fiber Composite



Stress-Strain Behavior of Aligned-Fiber Composite

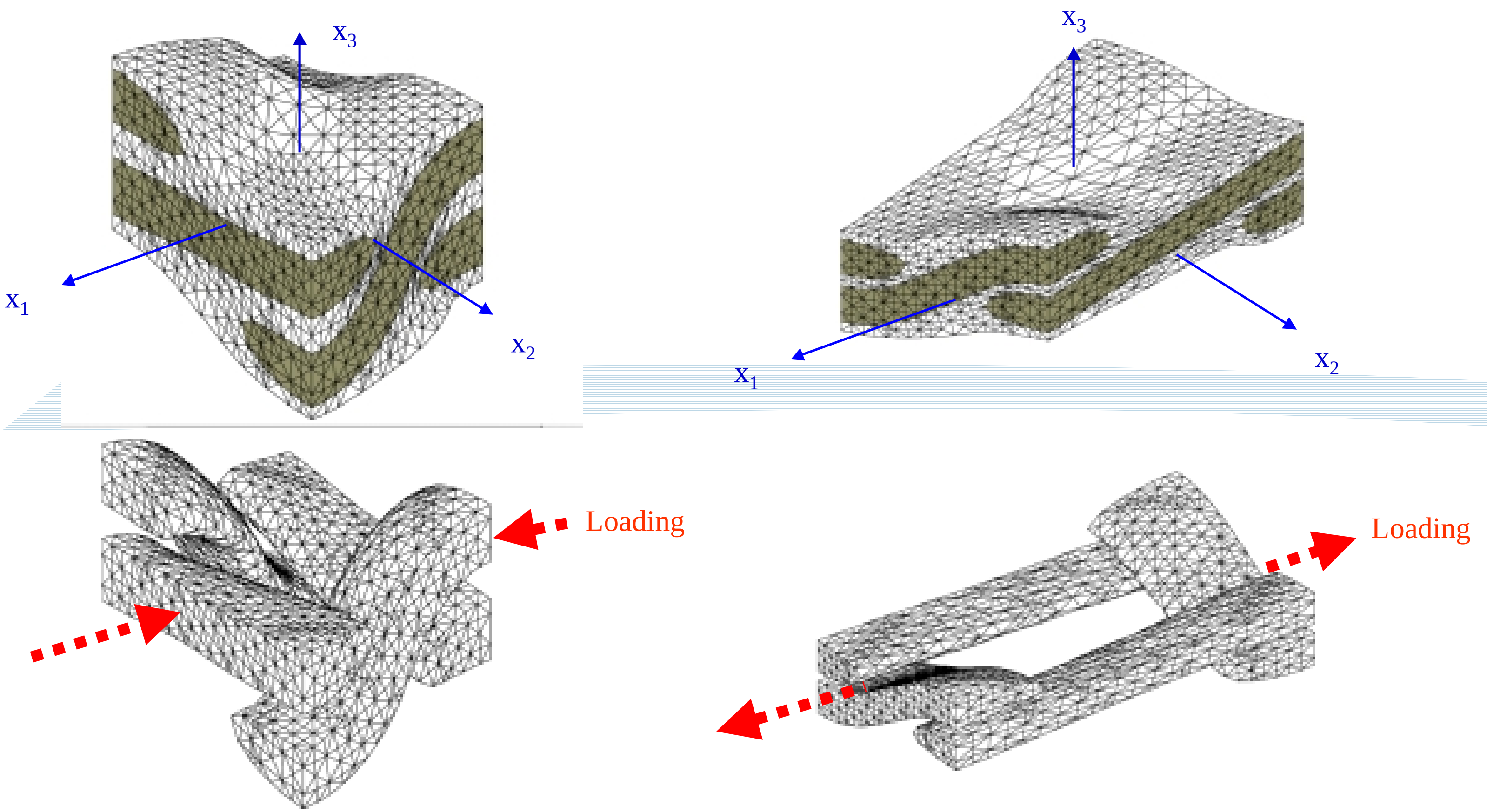


Unit Cell of Plain-Weave Textile Composite



(a) Unit Cell for Plain-Weave Textile Composite (b) Quarter of Unit Cell

Deformed Shapes of Plain-weave Textile

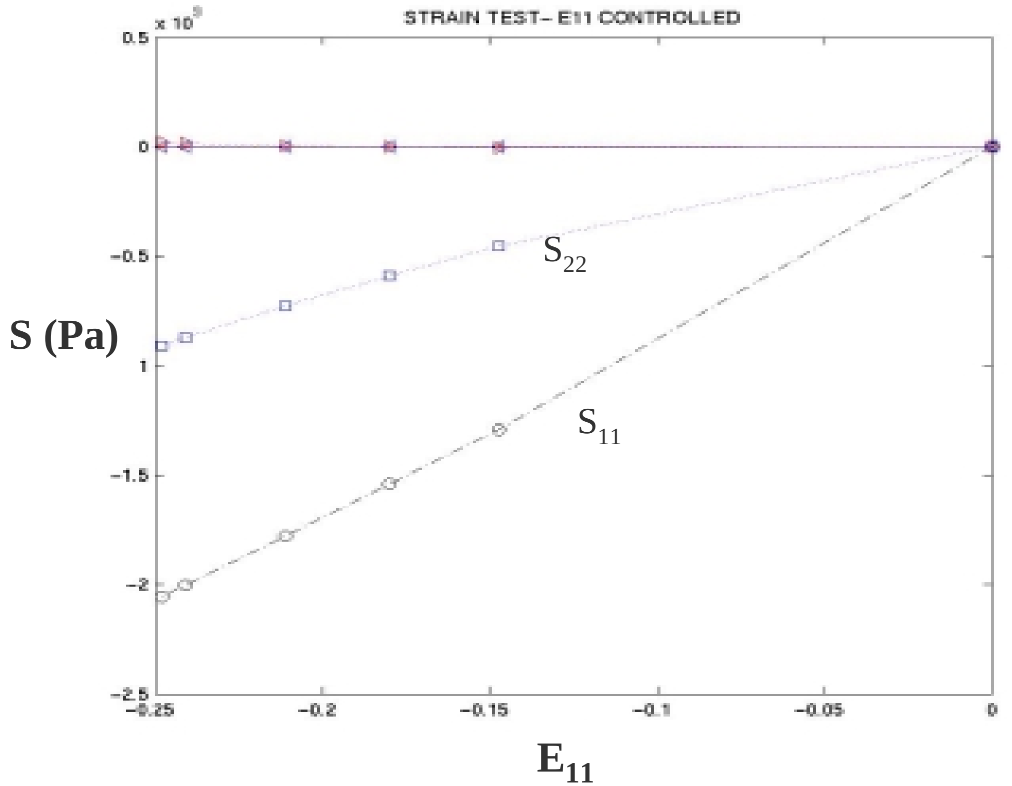


(a) uni-axial compression

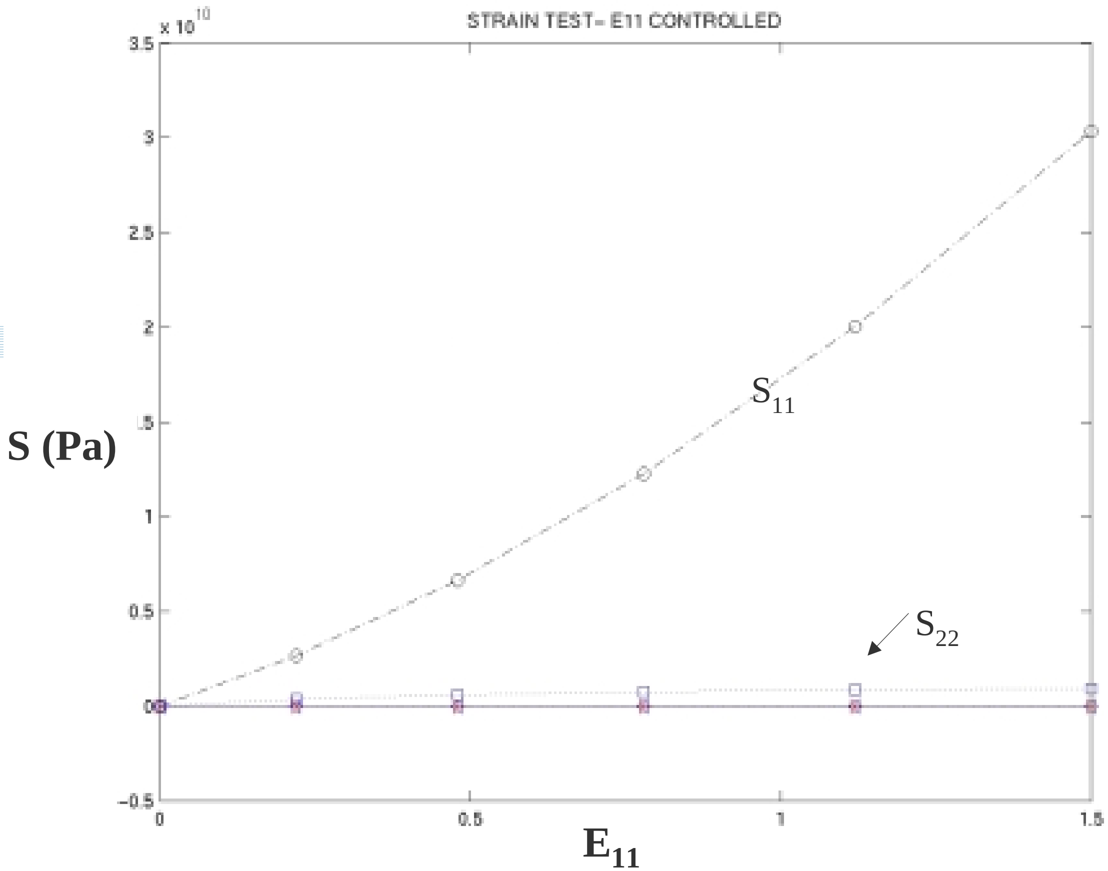
(b) uni-axial stretch

Stress-Strain Behavior of Plain-weave Textile Composite

(In-plane, uni-axial compression)



Stress-Strain Behavior of Plain-weave Textile Composite
(In-plane uni-axial stretch)



Summary

- Modeled finite deformation behavior at fiber diameter scale
- Performed hyperelastic modeling of aligned fiber and matrix yarns
- Computed onset of compression buckling in textile composite
- Computed strong tension-stiffening at large stretches

On-Going Issues

- Progressive material failure in yarns
  - fiber-matrix debonding
  - fiber breakage
  - matrix cracking

- Extended transversely isotropic hyperelastic-damage model for yarns accounting for failure effects noted above

- Development/calibration of orthotropic hyperelasticity model accounting for finite deformation effects in textile composite.

- Micro-mechanical modeling of yarn buckling in textile composites with linearized geometric stiffness and eigenmode analysis.