Multi-Scale Unit-Cell Analysis of Textile Composites' Finite Deformation Stiffness Characteristics

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Overview

1. Objectives: Material models for textile composites to facilitate structural analysis

   Enhanced understanding of textile composites' property-structure relations

2. Approach: Repeated unit-cell analysis and homogenization on multiple scales.

3. Results on fiber-diameter scale

4. Results on textile unit-cell scale

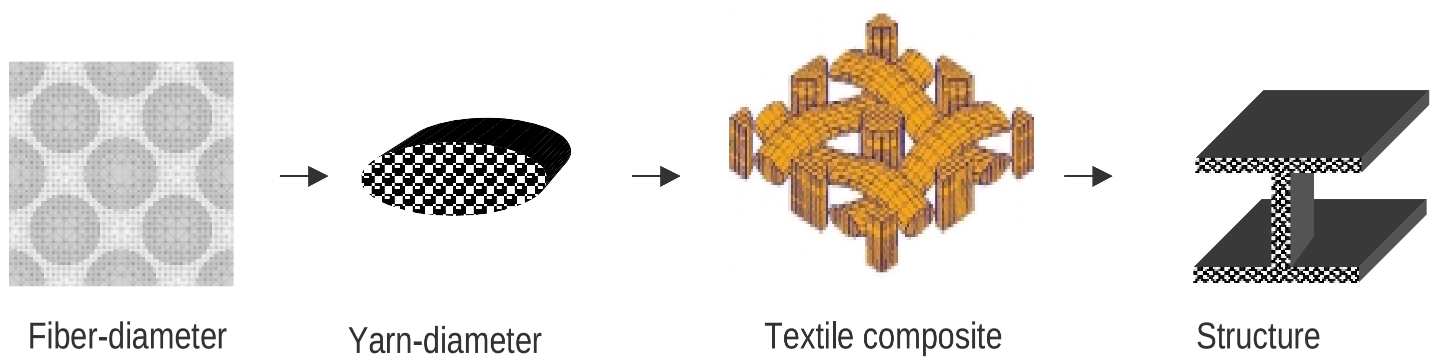
Research Objectives

- Identify suitable forms of constitutive models for textile composites

- Evaluate suitability and consistency of model forms using unit-cell analysis

- Better understand property-structure relations
  - specific interest: geometric stiffness effects
  - positive tension stiffening / negative compression stiffening

- Length scales considered

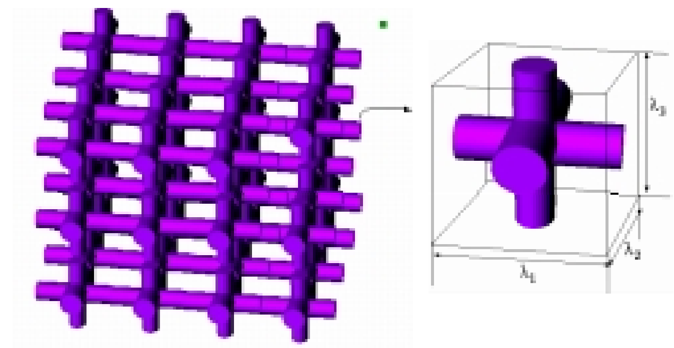


Essence of Unit-Cell Homogenization
(for heterogeneous, periodic media)

- **On a given length scale at which the material is heterogeneous (micro scale), apply an average stress or average deformation to a detailed model (unit cell)**
- **For each loading, compute detailed, equilibrium microscale stress and deformations fields.**
- **Take the spatial average of the “microscale” stress and deformation fields, to get their “macroscopic” correspondent.**
- **Develop/calibrate a constitutive model that adequately relates the macroscale stresses and deformations.**
- **When performing analysis of the system on the “macroscale” use the “homogenized” constitutive model to represent the medium.**

Micro-/Macro-scale Notation

- Periodic medium and unit cell



- Microscale stress/deformation

$$\boldsymbol{\sigma}(\mathbf{X}) = \boldsymbol{\Sigma} + \boldsymbol{\sigma}^*(\mathbf{X});$$

$$\mathbf{F}(\mathbf{X}) = \boldsymbol{\Phi} + \mathbf{F}^*(\mathbf{X});$$

$$< \boldsymbol{\sigma}^*(\mathbf{X}) > = \mathbf{0};$$

$$< \mathbf{F}^*(\mathbf{X}) > = \mathbf{0};$$

$$\mathbf{F}(\mathbf{X} + n\lambda_i \mathbf{e}_i) = \mathbf{F}(\mathbf{X}); \text{ periodicity of microscale deformation}$$

$$\boldsymbol{\sigma}(\mathbf{X} + n\lambda_i \mathbf{e}_i) = \boldsymbol{\sigma}(\mathbf{X}); \text{ periodicity of microscale stress}$$

- Averaging stress/deformation to find macroscale correspondents

$$\boldsymbol{\Sigma} = \langle \boldsymbol{\sigma} \rangle = \frac{1}{V} \int_{\Omega_s} \boldsymbol{\sigma} \, d\Omega_s;$$

$$\boldsymbol{\Phi} = \langle \mathbf{F} \rangle = \frac{1}{V} \int_{\Omega_s} \mathbf{F} \, d\Omega_s;$$

PROCESS: Deformation-Controlled Loading of Unit-Cell

- Specify an average state of deformation $\mathbf{\Phi}$ for the unit cell.
- Apply a consistent "homogeneous" displacement field $\mathbf{u} = \mathbf{\Phi}\cdot\mathbf{X}$ to unit cell.
- To achieve stress-field equilibrium on microscale, solve for the

  additive, periodic, heterogeneous displacement field $\mathbf{u}^*(\mathbf{X})$.
- Resulting equilibrium displacement field: $\mathbf{u}(\mathbf{X}) = \mathbf{\Phi}\cdot\mathbf{X} + \mathbf{u}^*(\mathbf{X})$
- For each macroscopic state of deformation $\mathbf{\Phi}$, compute the

  corresponding macroscopic state of stress $\mathbf{\Sigma}$.
- Consider the $\mathbf{\Sigma}$ versus $\mathbf{\Phi}$ behavior of the unit cell model.
- Provide and calibrate a macro-scale constitutive model $\mathbf{\Sigma} = \mathbf{\Sigma}(\mathbf{\Phi})$.

Symmetric, Conjugate, Macro Stress/Strain Measures

- Using conjugate macroscopic stress/strain measures ensures energy conservation between micro- and macro-scales.

- Nemat-Nassar (2000) demonstated/used conjugacy between macroscale deformation gradient Φ and the macroscale nominal stress <**P**>.

$$\left\langle \mathbf{P} : \dot{\mathbf{F}} \right\rangle = \left\langle \mathbf{P} \right\rangle : \left\langle \dot{\mathbf{\Phi}} \right\rangle$$

- It is preferred to develop constitutive models in terms of symmetric, macroscopic stress and deformation measures. Here, we use:

$$\hat{\mathbf{\Sigma}} = \left\langle \mathbf{P} \right\rangle \mathbf{\Phi}^{-\mathrm{T}};$$

$$\hat{\mathbf{E}} = \tfrac{1}{2}\left[\mathbf{\Phi}^{\mathrm{T}}\mathbf{\Phi} - \mathbf{I}\right];$$

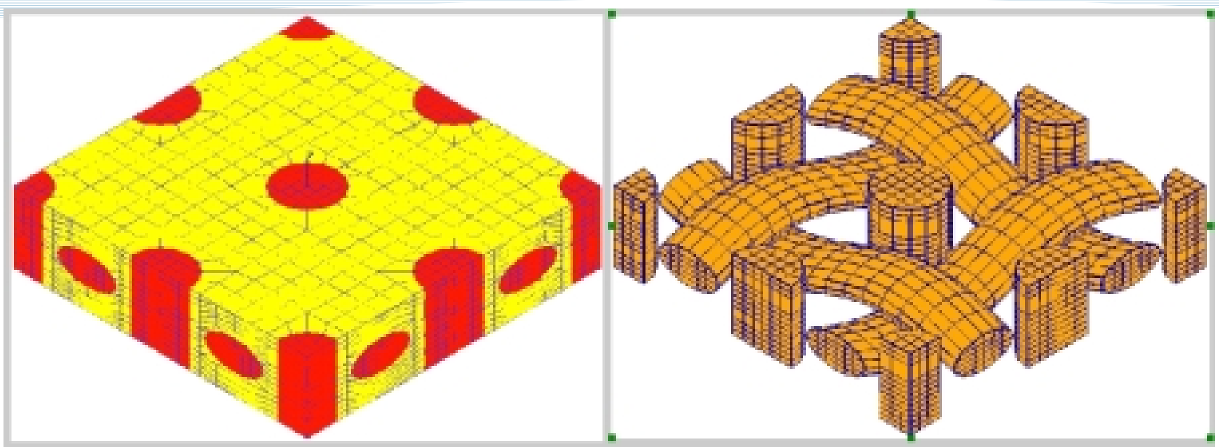
- These symmetric measures satisfy the following conjugacy relationship:

$$\hat{\mathbf{\Sigma}} : \dot{\hat{\mathbf{E}}} = \left\langle \mathbf{P} : \dot{\mathbf{F}} \right\rangle = \left\langle \mathbf{S} : \dot{\mathbf{E}} \right\rangle$$

Major Challenge

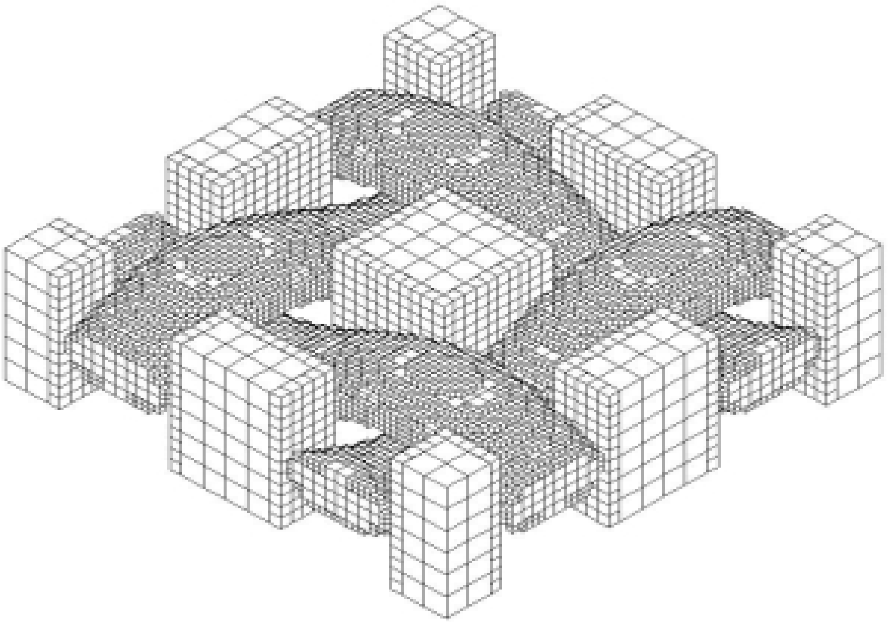
- Constructing appropriate unit cell models that capture:
    - material structure
    - material scale stress/strain fields

- Special issues pertaining to composites:
    - complex material interfaces
    - displacement periodicity (requires external face matching)
    - two-sidedness: meshing both sides of each material interface

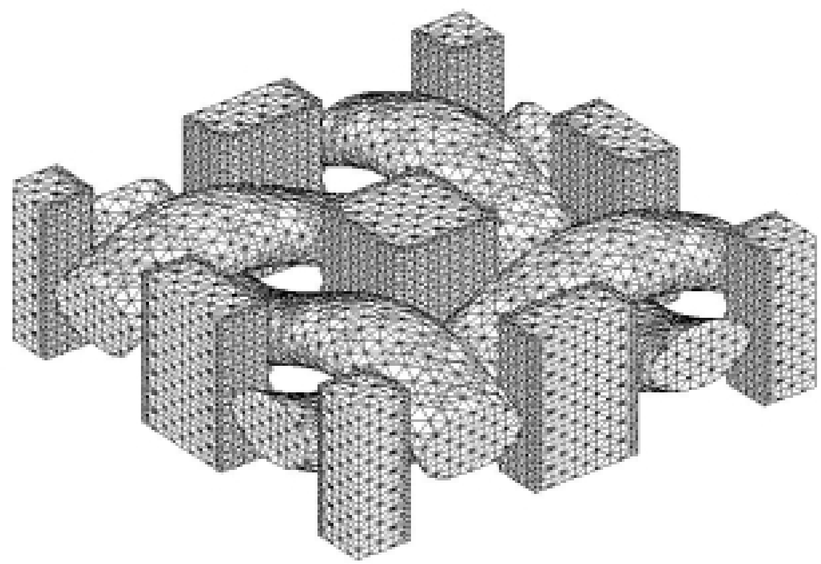


(a) unit-cell model for textile composite (b) woven yarn reinforcements

Alternative Unit-cell Meshing Approaches

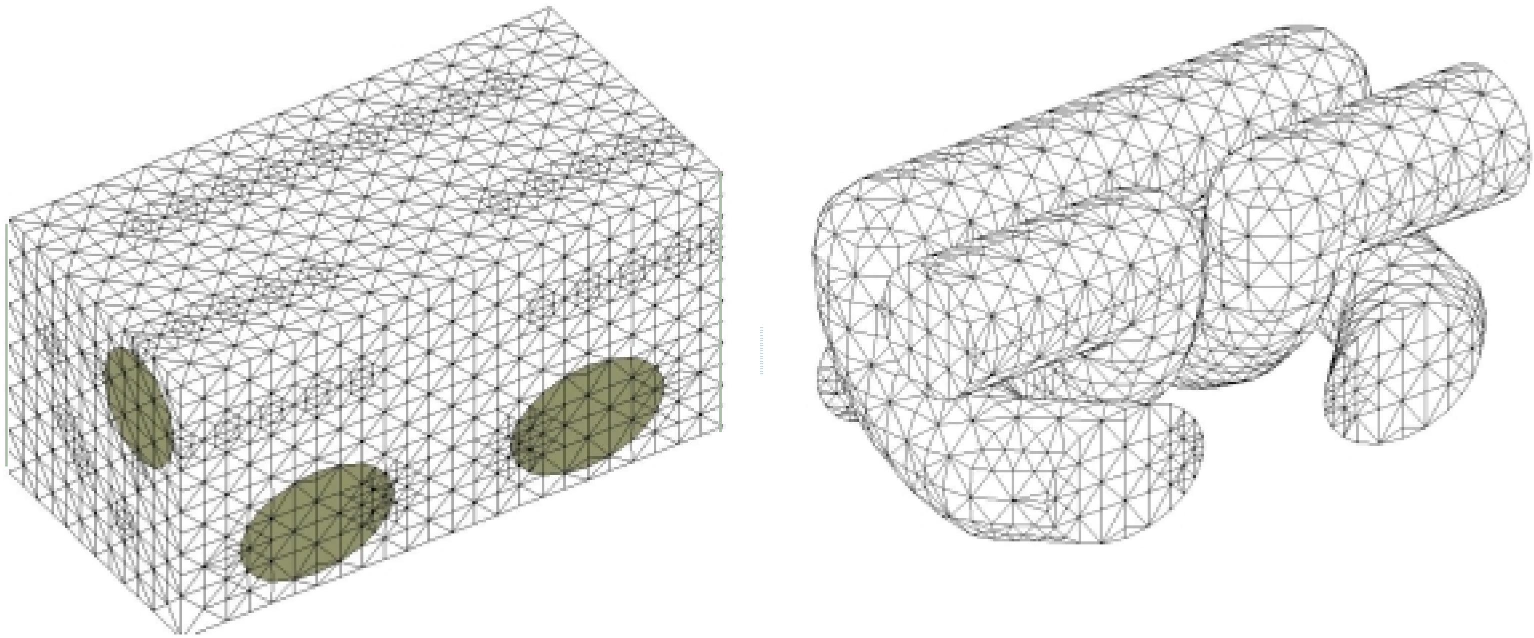


(a) voxel-based mesh



(b) tetrahedral mesh

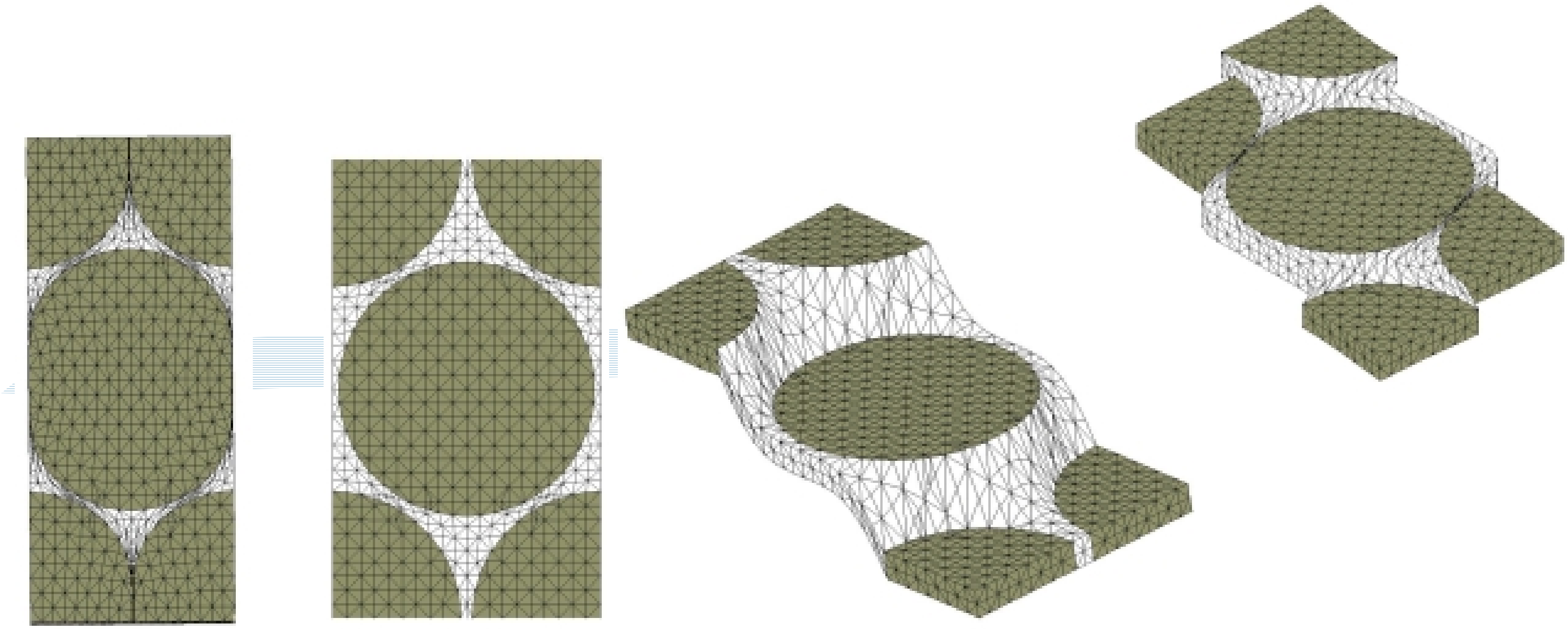
Unit Cell Model for Knitted Composite



(a) exterior of unit cell

(b) reinforcements only

Unit-Cell Analysis at Fiber-Diameter Scale



(a) In-plane compression (b) In-plane tension (c) Out-of-plane shear (d) In-plane shear

Transversely Isotropic Hyperelastic Model for Aligned Fiber Materials

- Model by Bonet and Burton (1998)

$$\psi = \psi(I_1, I_2, I_3, I_4, I_5) = \psi_{inh} + \psi_{ti}$$

$$\psi_{inh} = \tfrac{1}{2}\mu(I_1 - 3) - \mu \ln J + \tfrac{1}{2}\lambda(J - 1)^2$$

$$\psi_{ti} = [\alpha + \beta \ln J + \gamma(I_4 - 1)](I_4 - 1) - \tfrac{1}{2}\alpha(I_5 - 1)$$

where

$$I_1 = \mathrm{tr}(\mathbf{C}); \quad I_2 = \mathbf{C} : \mathbf{C}; \quad I_3 = \det(\mathbf{C}) = J^2;$$

$$I_4 = \mathbf{A} \cdot \mathbf{C} \cdot \mathbf{A}; \quad I_5 = (\mathbf{A} \cdot \mathbf{C}) \cdot (\mathbf{C} \cdot \mathbf{A})$$

and **A** is axis of symmetry or fiber director in undeformed configuration

Coefficient Estimation for Aligned-Fiber Composite

- Coefficient for Bonet and Burton's model $\boldsymbol{\alpha} = (\lambda, \mu, \alpha, \beta, \gamma)$
- Under Defomation-controlled homogenization

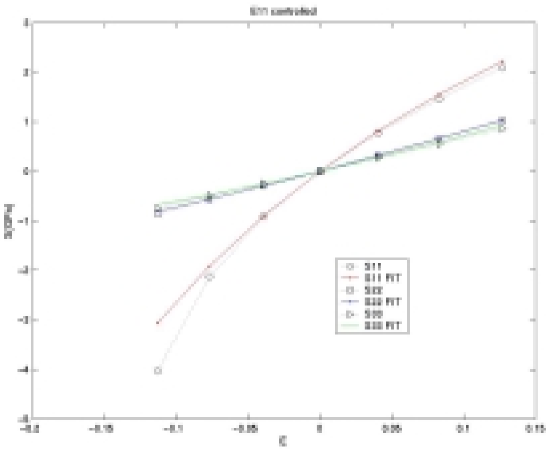
$$\min_{\boldsymbol{\alpha}} \sum_k \int_{t_1}^{t_2} \left\| \frac{\hat{\boldsymbol{S}}^k(\tau) - \tilde{\boldsymbol{S}}^k(\boldsymbol{\alpha}, \tau)}{\hat{\boldsymbol{S}}^k(\tau)} \right\| d\tau$$

where

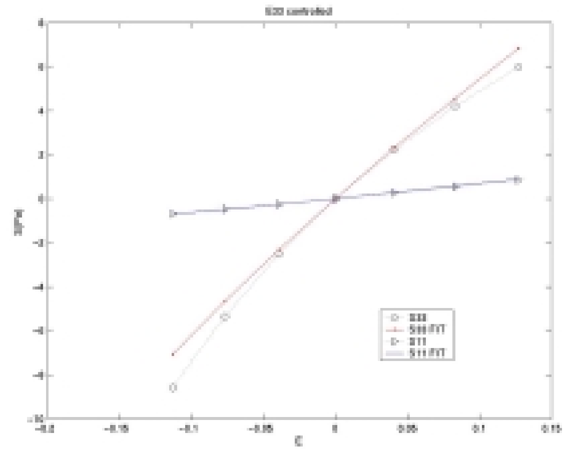
$\hat{\mathbf{S}}^k(\tau)$ is homogenized stress under k_{th} strain-controlled case

$\tilde{\mathbf{S}}^k(\boldsymbol{\alpha}, \tau)$ is second PK stress from proposed model.

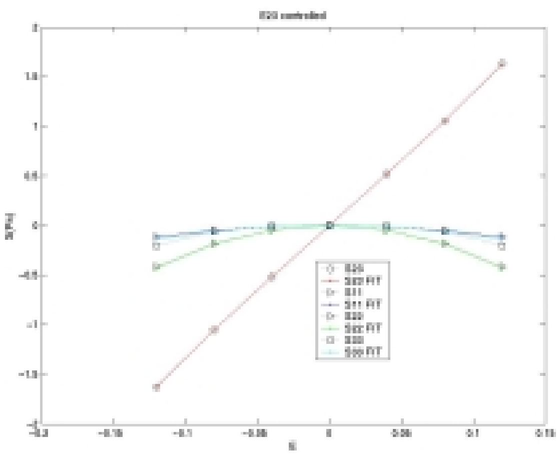
Homogenized, Macro-scale Stress-Strain Behavior of Yarn Composite



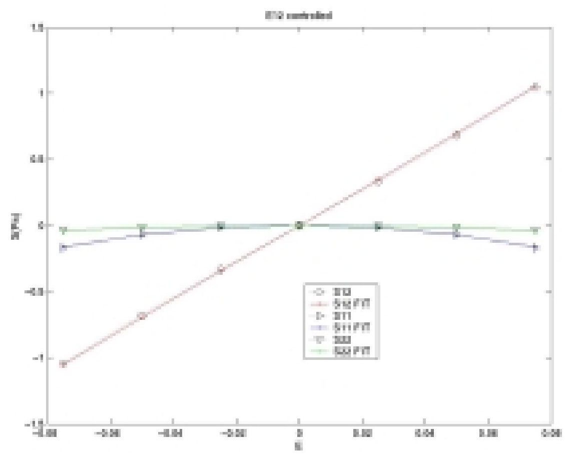
(a) $\Phi_{11} > 0$, others zero



(b) $\Phi_{33} > 0$, others zero

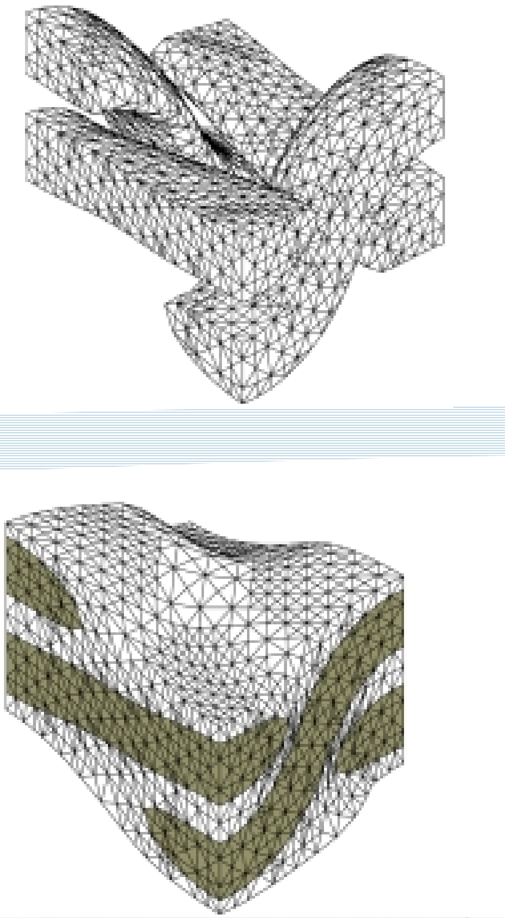


(c) $\Phi_{23} = \Phi_{32} > 0$, others zero

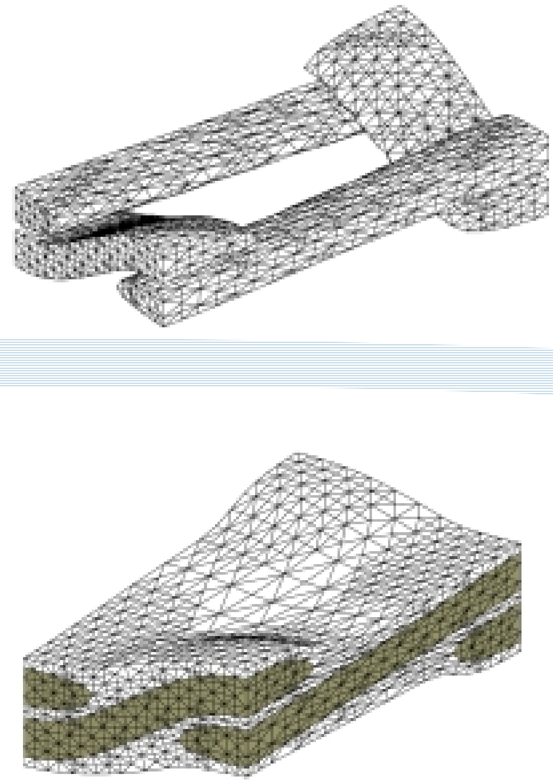


(d) $\Phi_{12} = \Phi_{21} > 0$, others zero

Analysis at the Textile Scale for Plain-Weaves

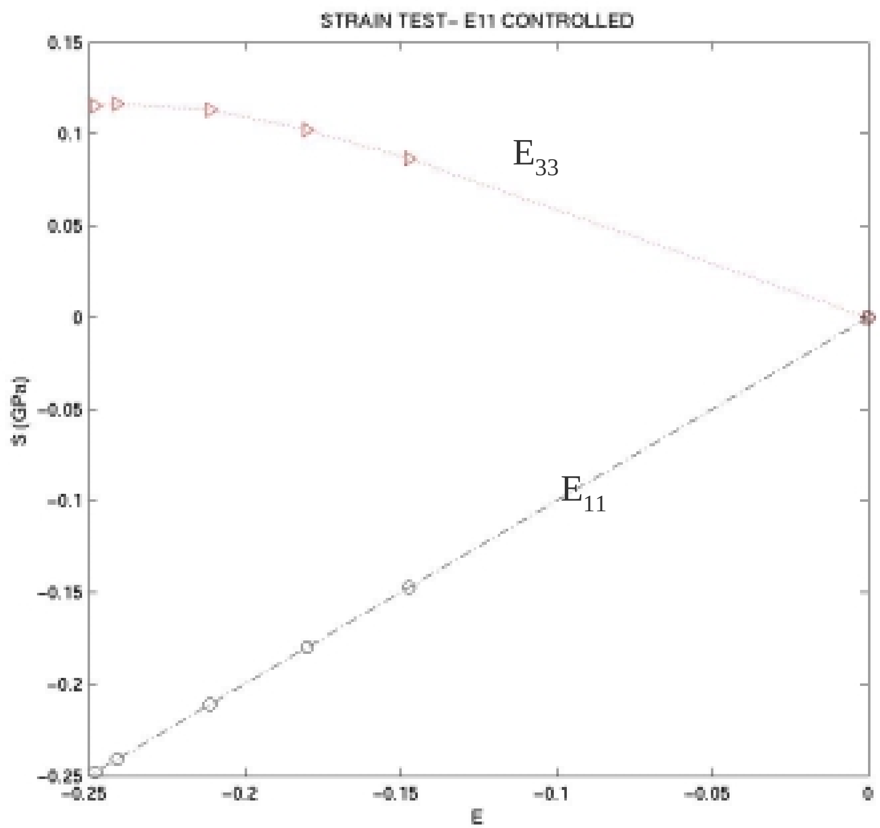


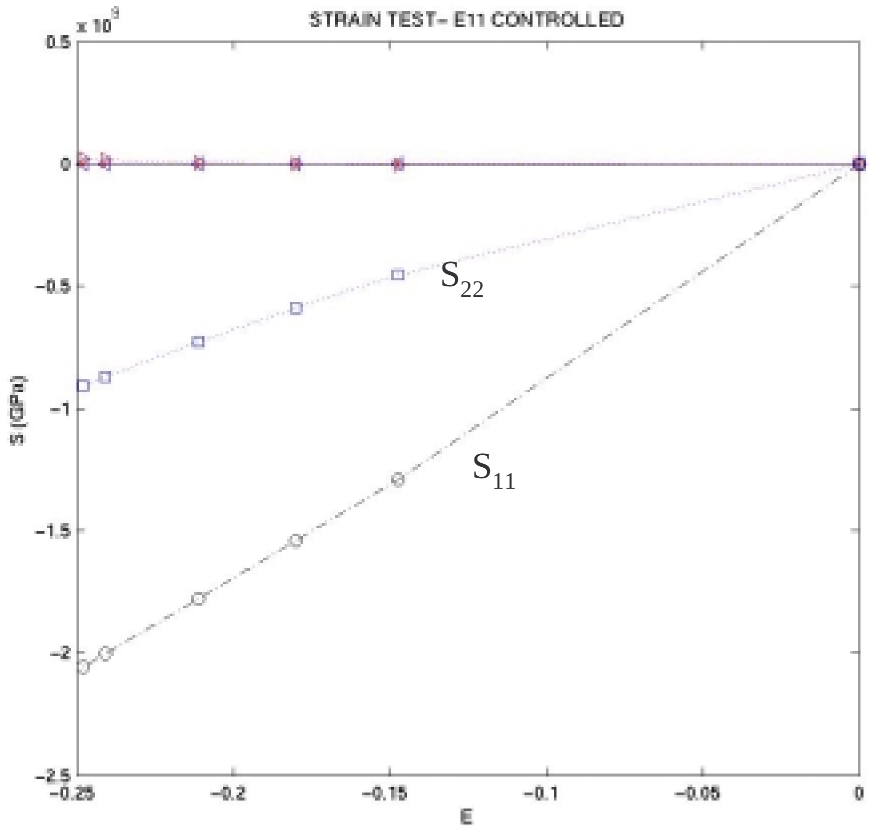
(a) uni-axial compression



(b) uni-axial tension

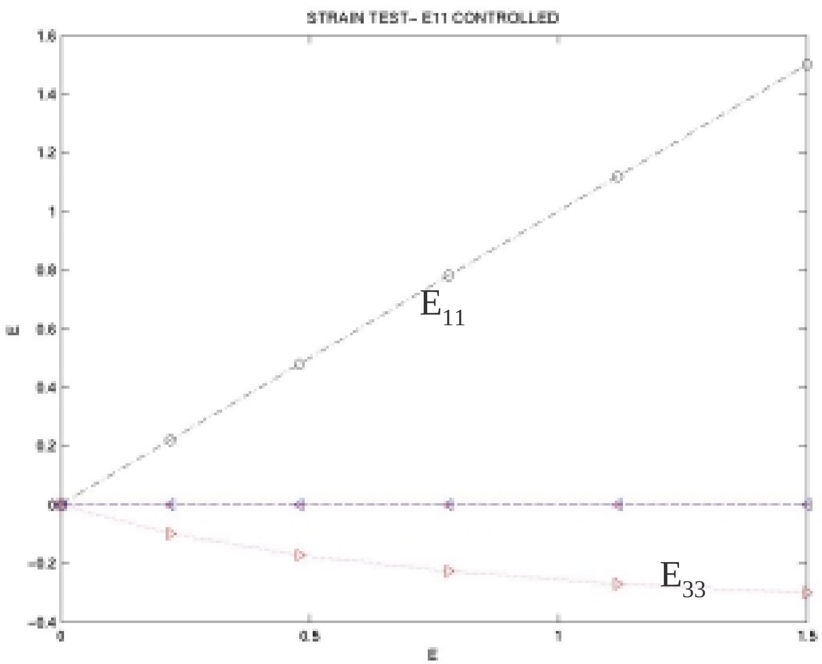
Stress-Strain Behavior of Plain-weave Textile Composite

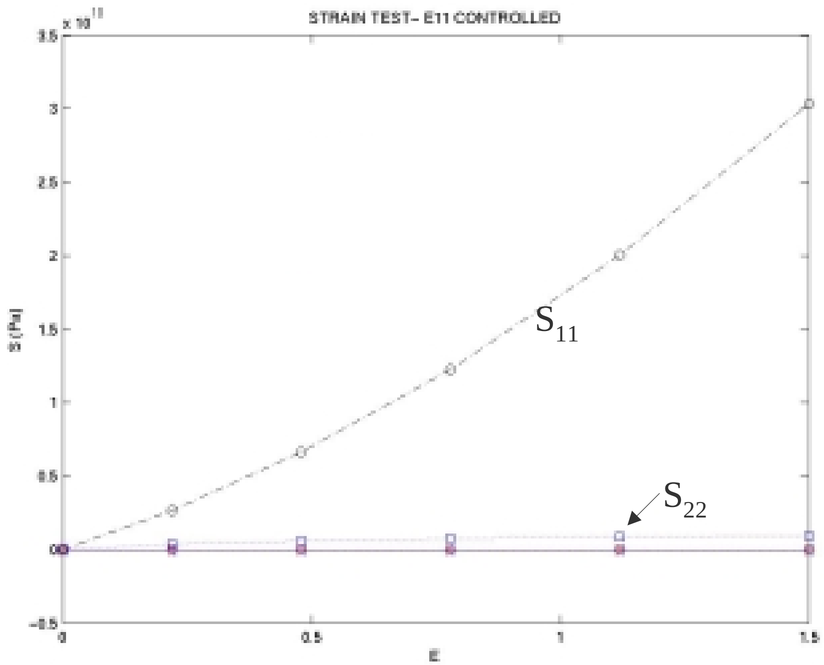




(a) uni-axial compression

Stress-Strain Behavior of Plain-weave Textile Composite (Continued)





(b) uni-axial tension

SUMMARY

- Have considered finite-deformation unit cell analysis of textile composites on multiple length scales.
- General Contributions:
  - **Development of automatic mesh generator suitable for most periodic composites.**
  - **Problem formulation at finite deformations**
    - **Symmetric, conjugate stress and strain measures**
- Hyperelastic analysis of plain-weave composite
  - **finite deformation at fiber diameter scale**
  - **material modeling of yarn composite**
  - **tendency for compression buckling and tension stiffening observed**

Extensions of Current Work

- Interface modeling with experimental testing program
  - **For example, if graded material properties at fiber-matrix interface are measured, incorporate these into models to capture their effect.**
- Material Failure Considerations (Fiber-diameter scale):
  - **incorporate and model effects of fiber breakage & matrix cracking**
- Material Modeling at Textile Scale
  - **suitable orthotropic hyperelastic model that captures compression buckling and tension stiffening effects**
  - **damage model to capture gross effects of fiber-diameter scale material failure**