Continuum Topology Optimization of Buckling-Sensitive Structures

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43rd AIAA SDM Conference
Denver, Colorado USA
24 April 2002

Presentation Overview

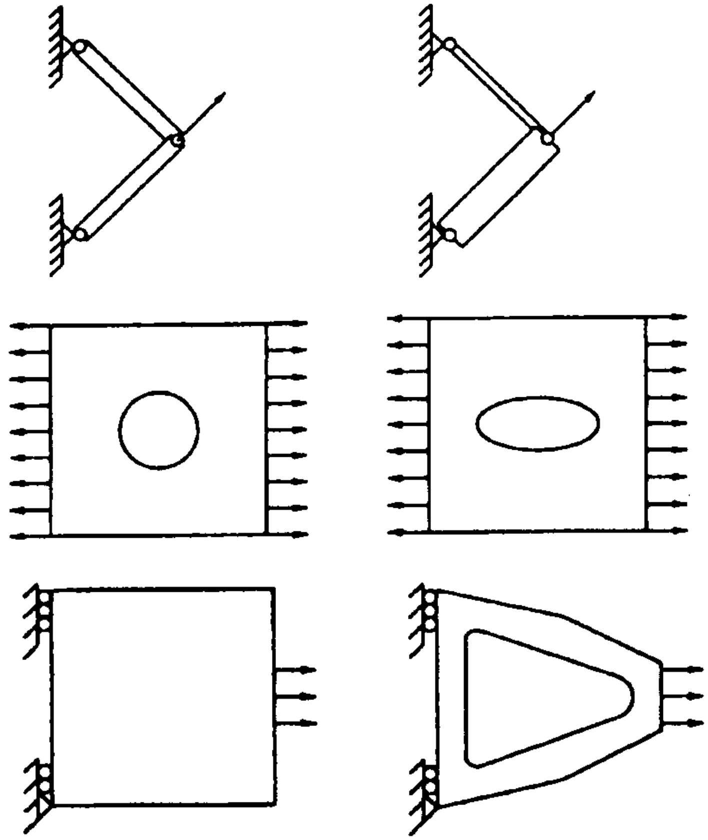
- Introduction to continuum structural topology optimization
- Alternative design-variable formulations
- Design-variable interpolation options
- Issues of structural sparsity and instability
- Problem formulations for sparse structures
- Analysis problem size reduction technique
- Examples
- Summary

What is “Structural Topology Optimization”

- Size Optimization

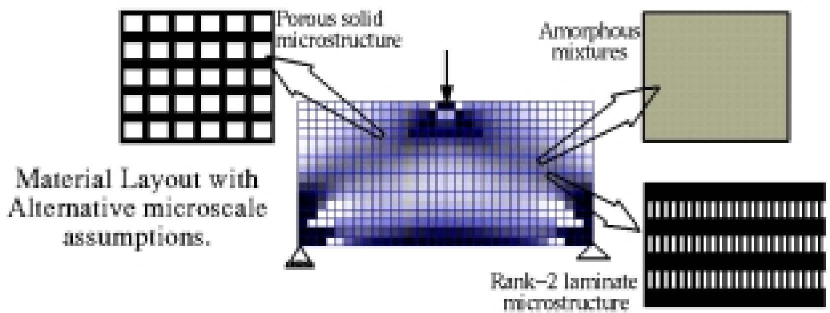
- Shape Optimization

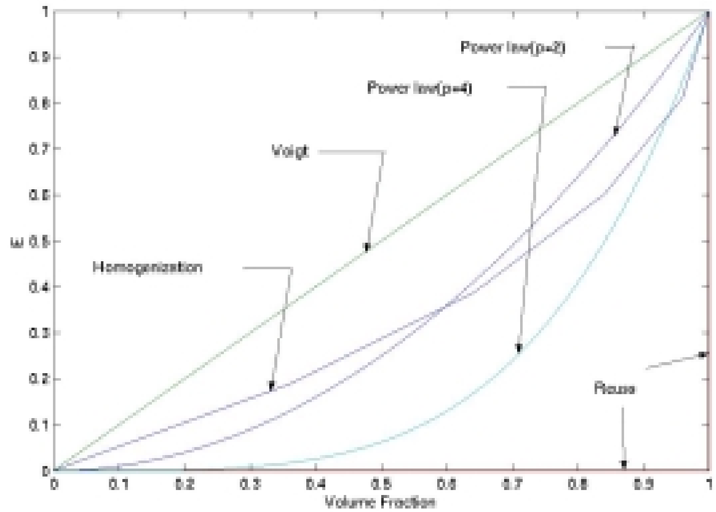
- Topology Optimization



Essentials of Continuum Structural Topology Optimization

- Discretization of structural domain Ω_d into a mesh of nodes/volumes.
- Use discretized model to describe spatial distribution of *design variables*.
- Specify a micro-mechanical model to relate local design variables to local mechanical properties.
- Pose and solve an optimization problem to extremize structural performance subject to material usage constraints.





The "Density" or "Volume-Fraction" Formulation

- Most widely used today
- Design variables: $\mathbf{b}=\{\phi_1, \phi_2, \phi_3, \ldots, \phi_n\}$

  where ϕ_i is either an element or node-based volume fraction of the structural material.

- Micro-mechanical (or mixing) rule:

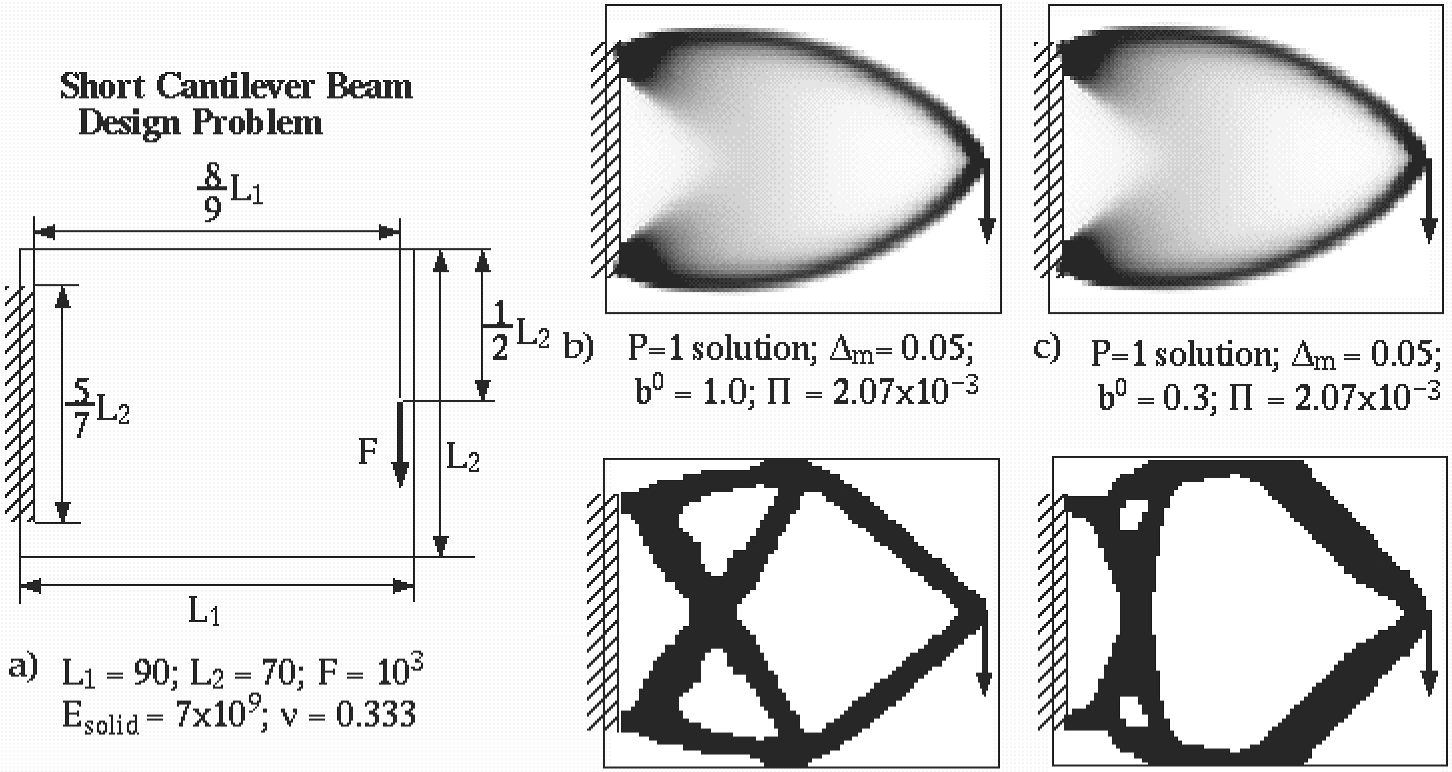
  $$\boldsymbol{\sigma}(\mathbf{X}) = \phi^p(\mathbf{X})\, \boldsymbol{\sigma}_{\mathbf{solid}}(\boldsymbol{\varepsilon}) + [1-\phi^p(\mathbf{X})]\, \boldsymbol{\sigma}_{\mathrm{void}}(\boldsymbol{\varepsilon})$$

  applies more easily to general elastic and inelastic material models than do micro-structure based formulations.

  Observation: The larger ***p***, the stronger the penalty against designs that utilize mixtures.

Characteristics of Alternative Formulations

Short Cantilever Beam Design Problem



a) $L_1 = 90$; $L_2 = 70$; $F = 10^3$
$E_{solid} = 7\text{x}10^9$; $\nu = 0.333$

b) P=1 solution; $\Delta_m = 0.05$;
$b^0 = 1.0$; $\Pi = 2.07\text{x}10^{-3}$

c) P=1 solution; $\Delta_m = 0.05$;
$b^0 = 0.3$; $\Pi = 2.07\text{x}10^{-3}$

d) Penalized sol.; $\Delta_m = 0.05$;
$b^0 = 1.0$; $\Pi = 3.16\text{x}10^{-3}$

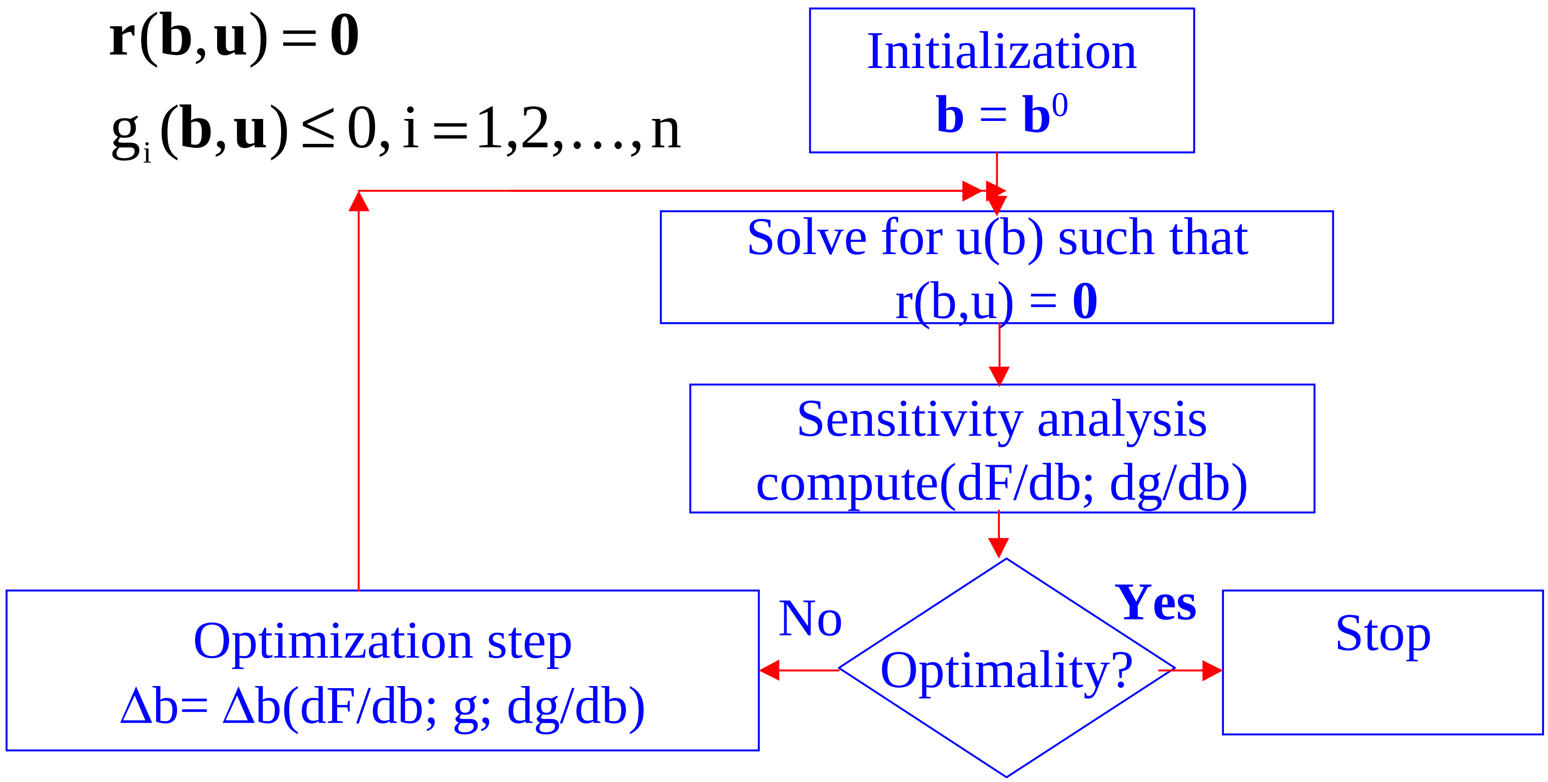
e) Penalized sol.; $\Delta_m = 0.05$;
$b^0 = 0.3$; $\Pi = 1.17\text{x}10^{-2}$

Design Optimization Algorithm to Solve:

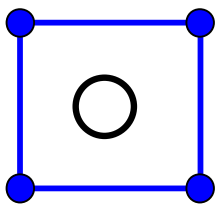
$$\min_{\mathbf{b}} F(\mathbf{b},\mathbf{u}) \text{ such that}$$

$$\mathbf{r}(\mathbf{b},\mathbf{u}) = \mathbf{0}$$

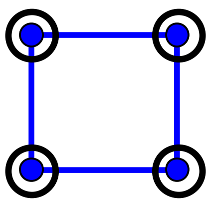
$$g_i(\mathbf{b},\mathbf{u}) \leq 0, i = 1,2,\ldots,n$$



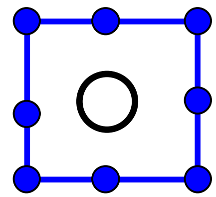
Displacement & Design Variable Interpolation Options



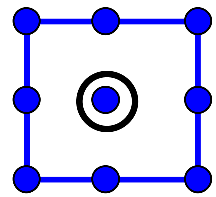
Q4/U: unstable



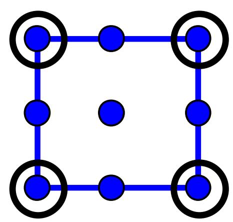
Q4/Q4: slightly unstable



Q8/U: Slightly unstable



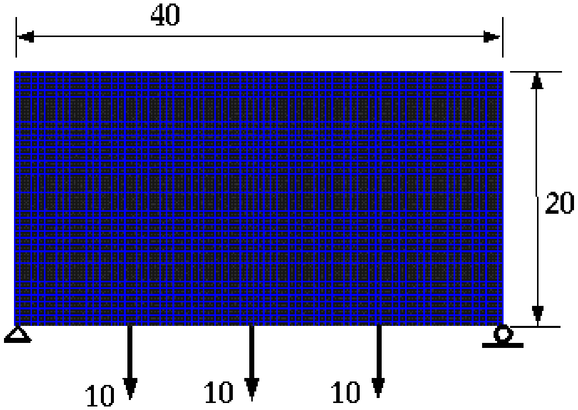
Q9/U: Slightly unstable

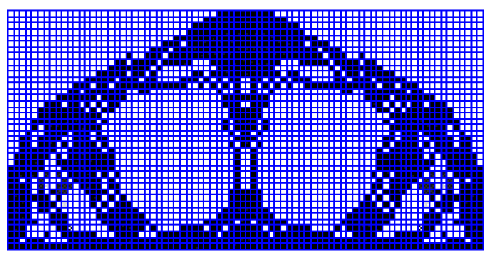


Q9/Q4: Stable

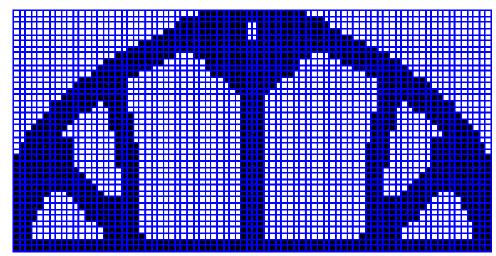
Established characteristics of continuum structural topology optimization as applied to civil structures

- The Q4/U element-based numerical formulation has severe instabilities that result in “checkerboarding” designs.
- As modeled, continuum structures are unrealistically “heavy”.
  - lack the sparsity of civil structures
  - continuum joints transmit moments
  - cannot capture potential buckling behaviours
- The optimization problem admits a large number of locally optimal design solutions.
- Here, we attempt to address the first two problems.





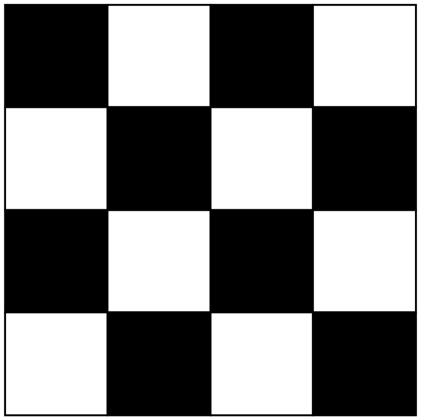
a) design featuring checkerboarding.



b) filtered design without checkerboarding.

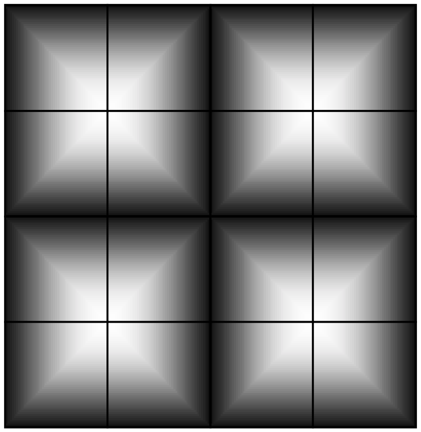
Element-based design variables (Q4/U):

- solid volume-fraction design field can be discontinuous across element boundaries.
- this can lead to the phenomenon of "checkerboarding" design solutions.
- checkerboarding designs can be eliminated via a number of ad-hoc spatial filtering techniques.

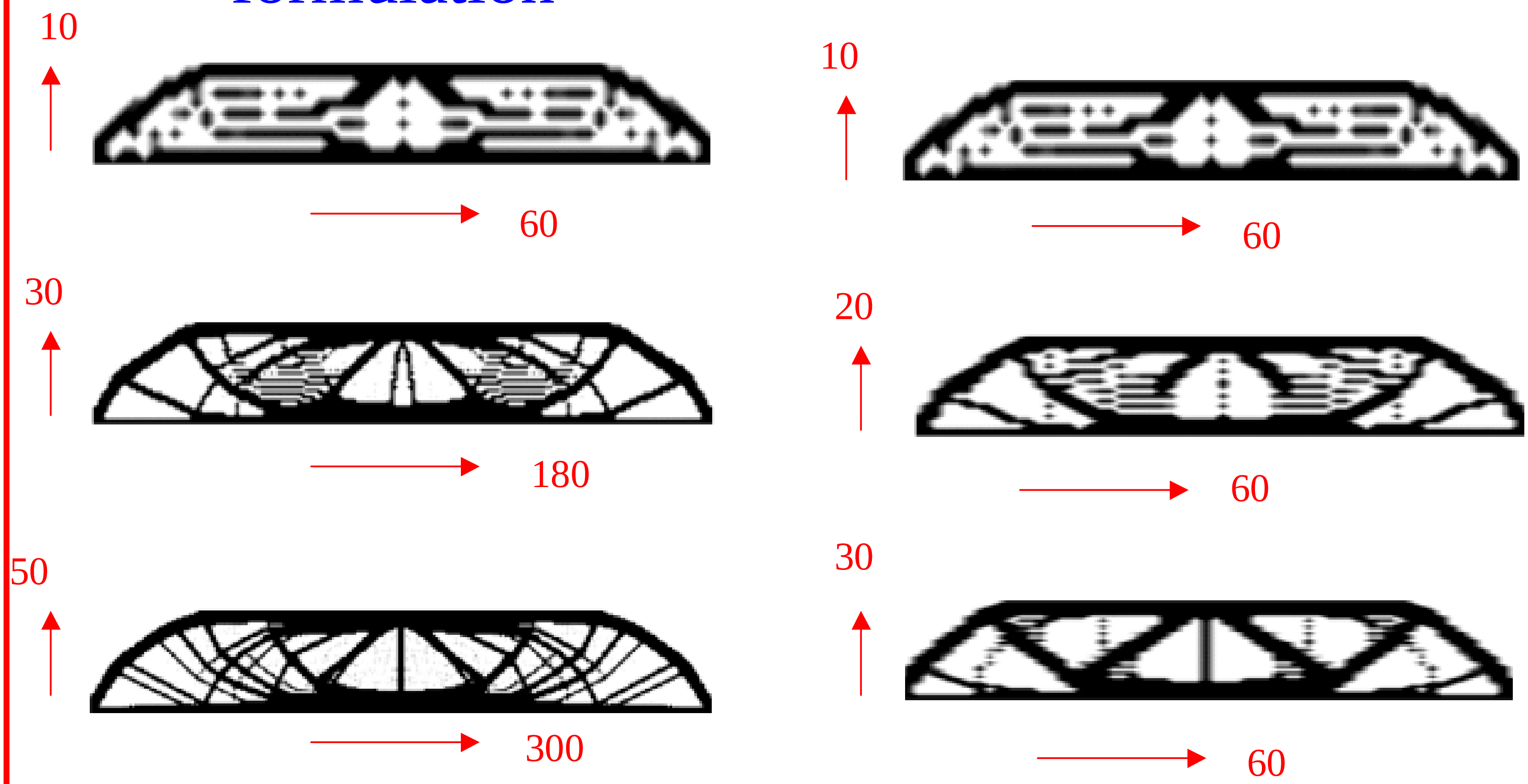


Node-based design variables (Q4/Q4):

- forces continuity of solid volume-fraction design variables across element boundaries.
- does not admit "checkerboarding" design solutions.
- requires no spatial filtering techniques.
- found by Jog & Haber (1996) to be slightly unstable



Layering instability appearing in Q4/Q4 formulation



Cure with uniform mesh refinement.

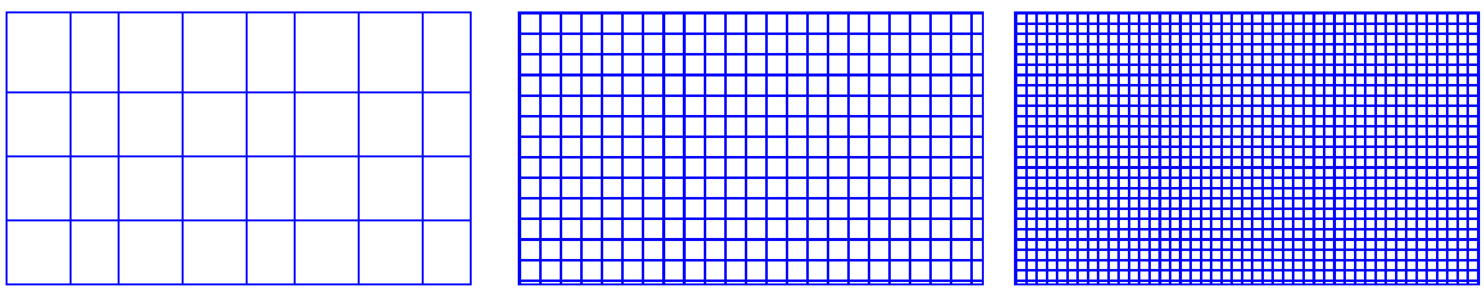
Cure with mesh refinement in direction of layering.

Structural Sparsity Issues

- Typically, large scale civil structures (as bridges) are sparse.
  - occupy only a small fraction of the structure's envelope volume.

- Structural models must capture the characteristic sparsity to yield realistic performance.

- This typically requires fine meshing of structural domain Ω_d.
  - This adds to the computational expense of the analysis and design process.



Alternative Design Formulations to Achieve Stable, Sparse Structures

- **<u>Option 1</u>:**
  - **Perform structural analysis considering geometrically nonlinear behaviors and instabilities.**
  - **Design/Optimize the structure to avoid instabilities.**
- **<u>Option 2</u>:**
  - **Perform linearized buckling stability analysis on the structure.**
  - **Design/Optimize to maximize minimum critical buckling load.**

Option 1: Analysis/optimization of the structure as a nonlinear hyperelastic system.

Constitutive law

$$U(J)=\frac{1}{2}K[\frac{1}{2}(J^2-1)-\ln(J)] \qquad \bar{W}=\frac{1}{2}\mu[tr(\bar{\boldsymbol{\theta}})-3]. \qquad \tau=JU'(J)\mathbf{1}+2dev\frac{\partial\bar{W}}{\partial\boldsymbol{\theta}}.$$

Variational formulation

$$\int_{\Omega_S}\tau_{ij}\delta\varepsilon_{ij}d\Omega_S=\int_{\Omega_S}\rho_o\gamma_j\delta u_j d\Omega_S+\int_{\Gamma_h}h_j\delta u_j d\Gamma_h \qquad d(\delta W^{\text{int}})=\int_{\Omega_S}\delta\varepsilon_{ij}dL_v(\tau_{ij})d\Omega_S+\int_{\Omega_S}\delta\varepsilon_{ji}\tau_{im}d\varepsilon_{jm}d\Omega_S,$$

Discrete Equilibrium

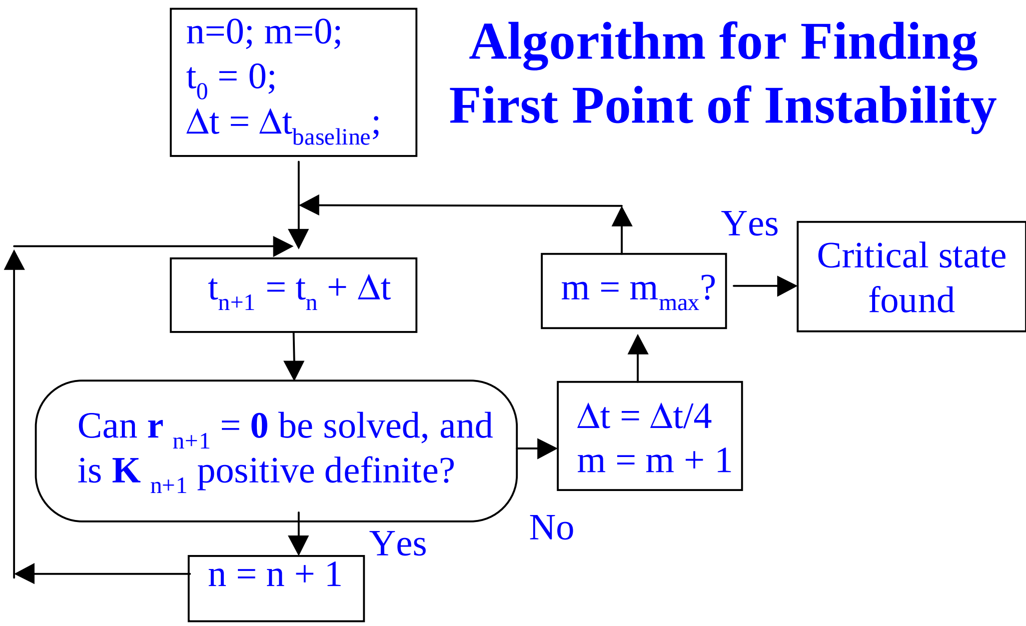
$$\boldsymbol{r}_{n+1}^A=(\boldsymbol{f}^{\text{int}})_{n+1}^A-(\boldsymbol{f}^{ext})_{n+1}^A=0 \qquad (\boldsymbol{f}^{\text{int}})_{n+1}^A=\int_{\Omega_S}(\boldsymbol{B}^A)^T:\boldsymbol{\tau}_{n+1}d\Omega_S \qquad (\boldsymbol{f}^{ext})_{n+1}^A=\int_{\Omega_S}\rho_o N^A\gamma_{n+1}d\Omega_S+\int_{\Gamma_h}N^A\boldsymbol{h}_{n+1}d\Gamma_h.$$

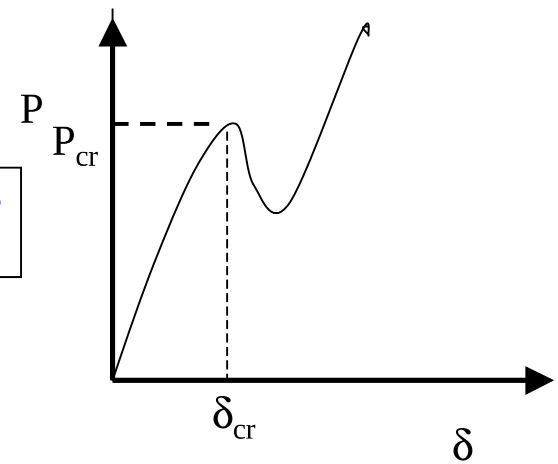
$$\boldsymbol{K}_{il}^{AB}=\int_{\Omega_s}B_{ji}^A c_{jk}B_{kl}^B d\Omega_s+\int_{\Omega_s}N_j^A\tau_{jk}N_k^B\delta_{il}d\Omega_s,$$

Option 1 (continued):

Solve for the Minimum Critical Instability Load

Algorithm for Finding First Point of Instability





Option 1: (Continued)

- Objective function:
  - $\boldsymbol{F} = (f_{crit})^{-1}$
- Once the minimum point of instability is found:
  - compute design derivative of minimum critical load.

$$\mathrm{f}_{\mathrm{crit}}^{\mathrm{int}} = \sum_{K \in \{n_g\}} \left[\frac{-\mathbf{g}_K}{\|\mathbf{g}_K\|} \cdot \int_{\Omega} \mathbf{B}_K^T \cdot \boldsymbol{\tau} \, d\Omega \right]$$

$$\frac{\mathrm{df}_{\mathrm{crit}}^{\mathrm{int}}}{\mathrm{d}\mathbf{b}} = \sum_{K \in \{n_g\}} \left[\frac{-\mathbf{g}_K}{\|\mathbf{g}_K\|} \cdot \int_{\Omega} \mathbf{B}_K^T \cdot \frac{\partial \boldsymbol{\tau}}{\partial \mathbf{b}} d\Omega + \left(\mathbf{u}_K^a\right) \cdot \frac{\partial \mathbf{r}}{\partial \mathbf{b}} \right]$$

$$\mathbf{K} \cdot \left(\mathbf{u}_K^a\right) = -\frac{\partial \left(\mathrm{f}_{\mathrm{crit}}^{\mathrm{int}}\right)_K}{\partial \mathbf{u}}$$

Option 2: Structure modeled as linear elastic with instability computed by linearized buckling analysis

Linear elastic problem:

$$\mathbf{K}_L \cdot \mathbf{u} = \mathbf{f}^{ext}$$

Associated generalized eigenvalue problem:

$$\mathbf{K}_L(\mathbf{b})\boldsymbol{\psi} + \lambda \mathbf{G}(\mathbf{u},\mathbf{b})\boldsymbol{\psi} = \mathbf{0}; \qquad \lambda = -\frac{\boldsymbol{\psi}^T \mathbf{K}_L \boldsymbol{\psi}}{\boldsymbol{\psi}^T \mathbf{G} \, \boldsymbol{\psi}}$$

Modified eigenvalue problem

$$\left[\left(\mathbf{K}_L + \mathbf{G}\right) + \gamma \mathbf{K}_L\right] \cdot \boldsymbol{\psi} = \mathbf{0}; \qquad \left(\gamma = \frac{\lambda - 1}{\lambda}\right)$$

Objective function and design derivative

$$\mathrm{F}(\mathbf{u},\mathbf{b}) = \frac{1}{\min(\lambda)};$$

$$\frac{\mathrm{dL}}{\mathrm{d}\mathbf{b}} = -\boldsymbol{\psi}^T \left(\frac{\partial \mathbf{G}}{\partial \mathbf{b}} + \frac{1}{\lambda} \frac{\partial \mathbf{K}_L}{\partial \mathbf{b}} \right) \boldsymbol{\psi} + (\mathbf{u}^a)^T \left(\frac{\partial \mathbf{K}_L}{\partial \mathbf{b}} \cdot \mathbf{u} - \frac{\partial \mathbf{f}^{ext}}{\partial \mathbf{b}} \right)$$

Potential Problem in Structural Analysis with Option 1:

- **Elements devoid of structural material are highly compliant.**

- **These elements can undergo excessive deformation creating difficulty/singularity in solving the structural analysis problem.**

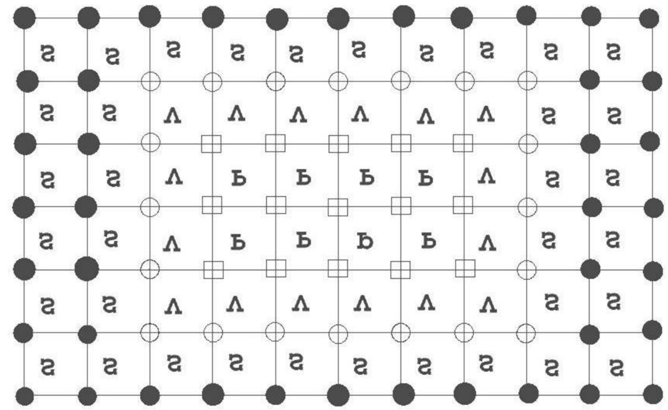
- **A strategy to circumvent this problem is needed.**

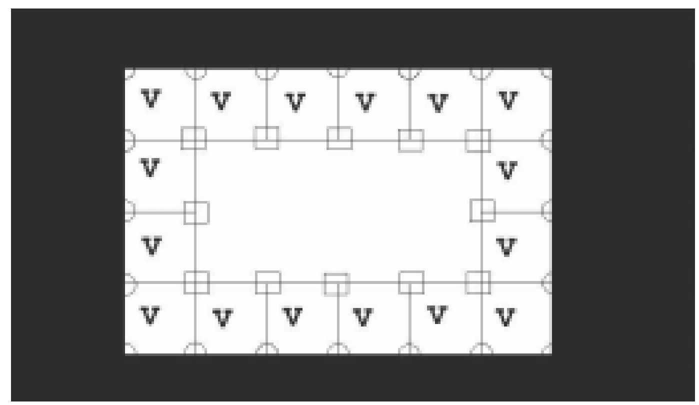
Analysis Problem Size Reduction Technique

Identify void elements

Identify and restrain "prime" nodes (those surrounded by void elements).

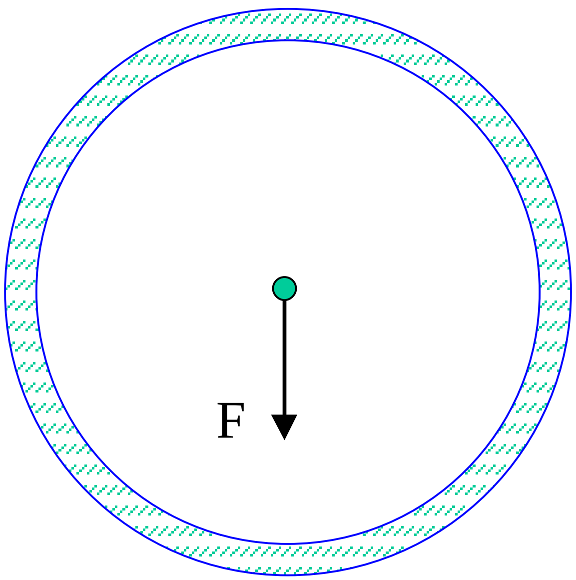
Identify and temporarily remove "prime" elements (those surrounded only by prime nodes).

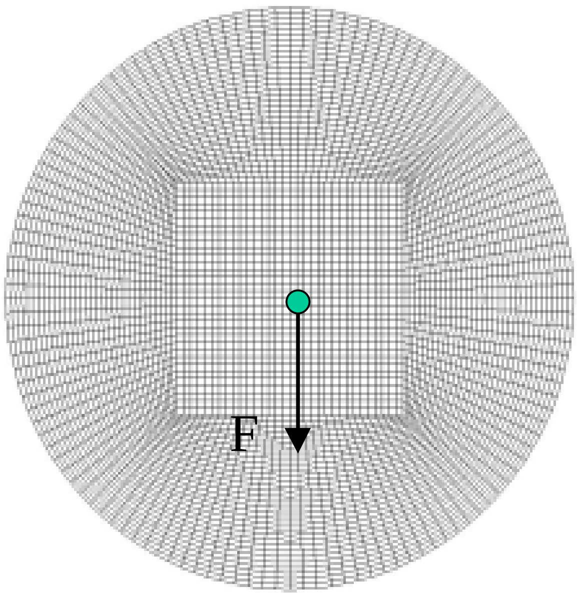




Test Problem #1 for Design of Sparse Buckling Sensitive Structure.

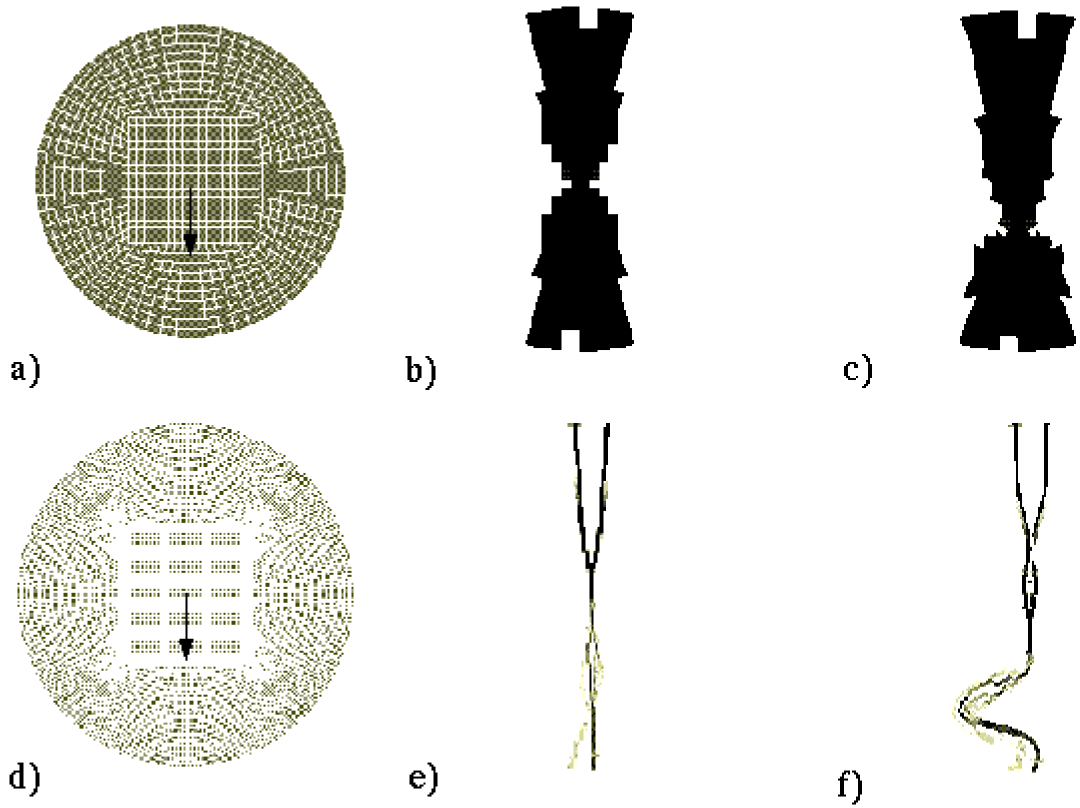
• Design optimum sparse, elastic structure in the circular domain to carry the design load back to fixed, rigid walls.



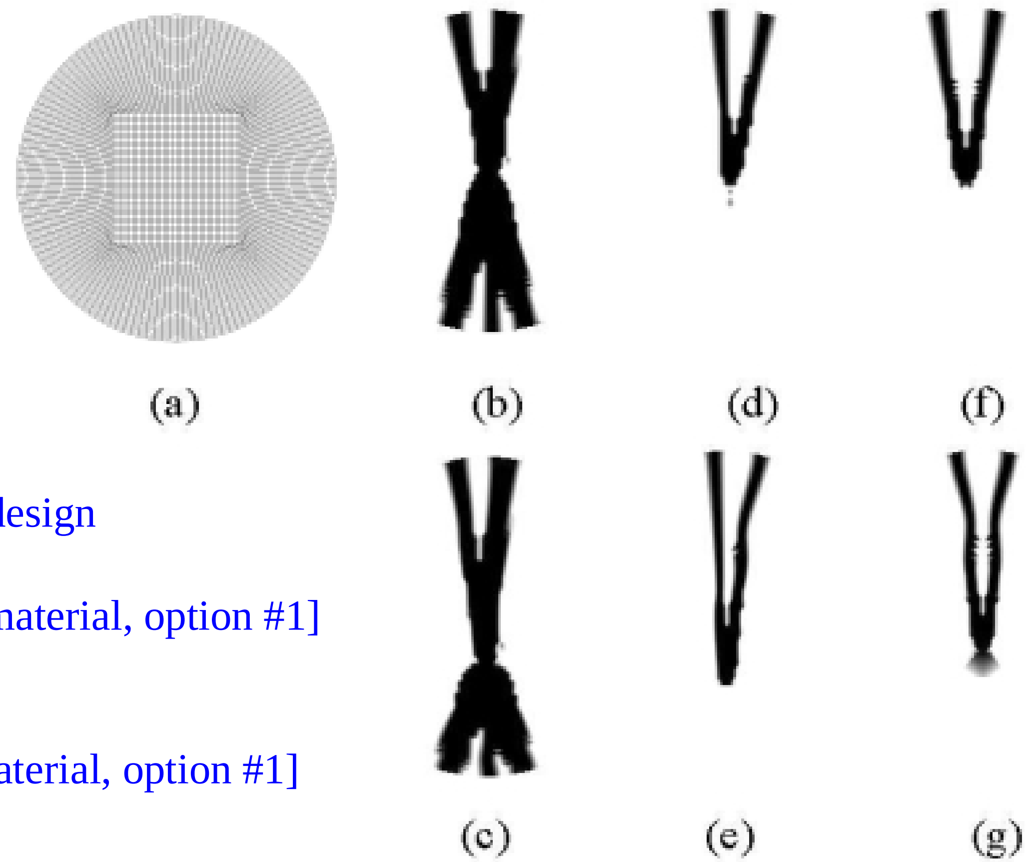


Old Q4/U Design Solutions to Circle Problem

- Generalized compliance objective function w/ spatial filtering



New Q4/Q4 Solutions to Circle Test Problem:



(a) Mesh used in analysis/design

(b) Design solution [20% material, option #1]
(c) Deformed shape of (b)

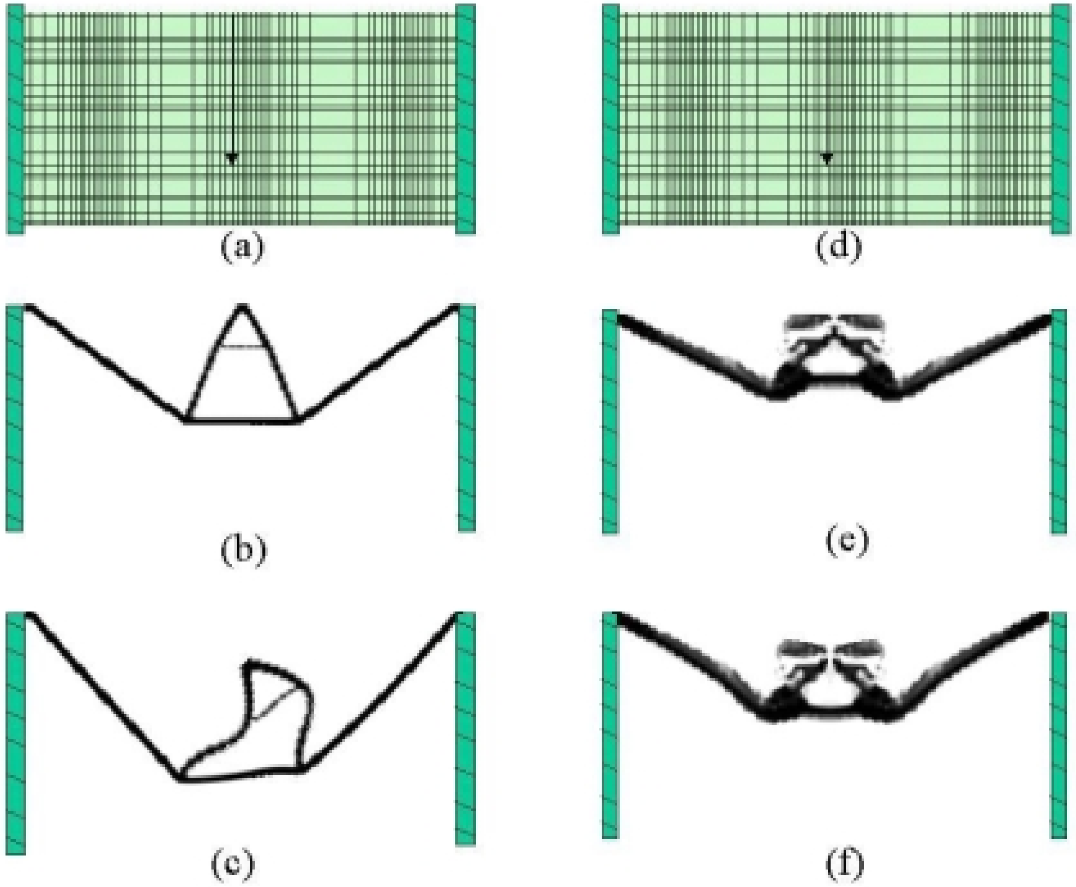
(d) Design solution [5% material, option #1]
(e) Deformed shape of (d)

(f) Design solution [5% material, option #2]
(g) Deformed shape of (f)

Fix-end Beam Problem

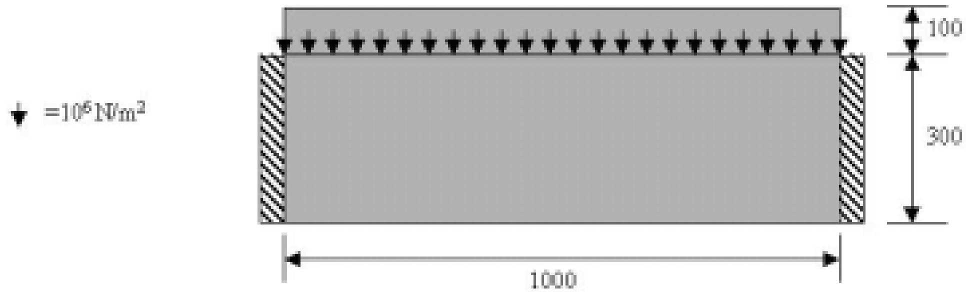
Option #1:
(a) Mesh, restraints, load case
(b) Design solution
(c) Design at onset of instability

Option #2:
(d) Mesh, restraints, load case
(e) Design solution
(f) Buckling mode (local)

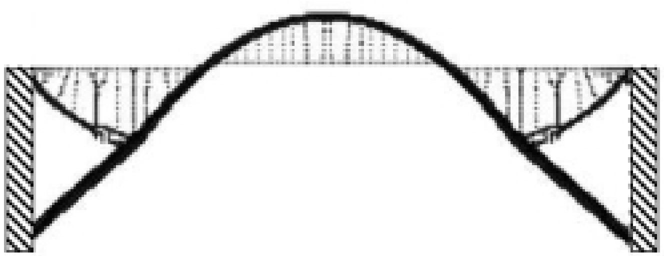


Sparse, Long Span Canyon Bridge Concept Design Problem:

- span = 1000m; max height = 400m;
- design can occupy only 10% of envelope volume
- 16000 elements, 16281 design variables used.

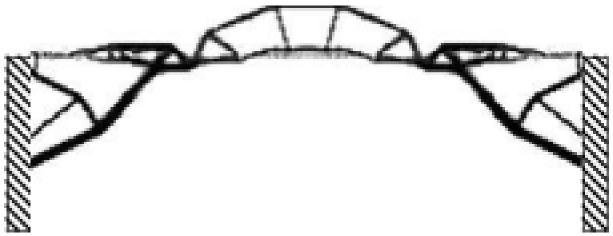


Problem description



minimizing linear elastic compliance

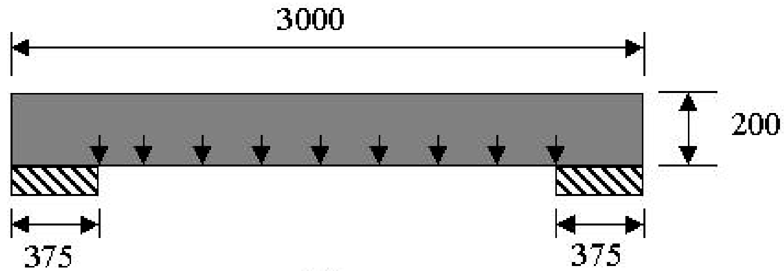
$\Pi = 2.3 \cdot 10^7;\ \lambda = 8.3 \cdot 10^1$



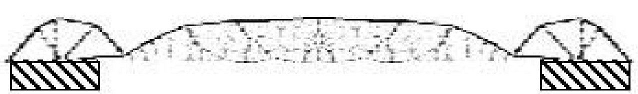
maximizing the minimum buckling load

$\Pi = 6.5 \cdot 10^7;\ \lambda = 4.9 \cdot 10^3$

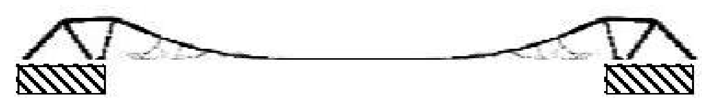
Design of Long-Span Over-Deck Bridge



(a)



(b)



(c)

Design to minimize elastic compliance

$\Pi = 1.65;\ \lambda = 3.2 \cdot 10^2$

Design to maximize minimum linearized buckling eigenvalue

$\Pi = 8.20;\ \lambda = 1.5 \cdot 10^4$

Summary and Conclusions

- For continuum structural topology optimization methods to be useful in concept design of large-scale civil structures, they must be able to detect potential buckling instabilities.
    - modeling of sparse structures at high mesh refinement.
    - solution of structural analysis problems with geometric nonlinearity.
- The proposed formulations are promising for concept design of stable, sparse structural systems.