Comparison of Continuum and Ground Structure Topology Optimization Methods for Concept Design of Structures

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FACT:

- Techniques for both continuum and discrete ground−structure topology optimization have been actively investigated over the past two decades.

QUESTIONS:

- Is either method clearly superior to the other for design of large−scale civil engineering structures?

- What are the relative strengths/weaknesses of the two approaches?

Presentation Overview

- Brief summary of continuum and discrete formulations.
- Comparative solutions of a truss design problem.
- Observations and additional issues.
- Conclusions

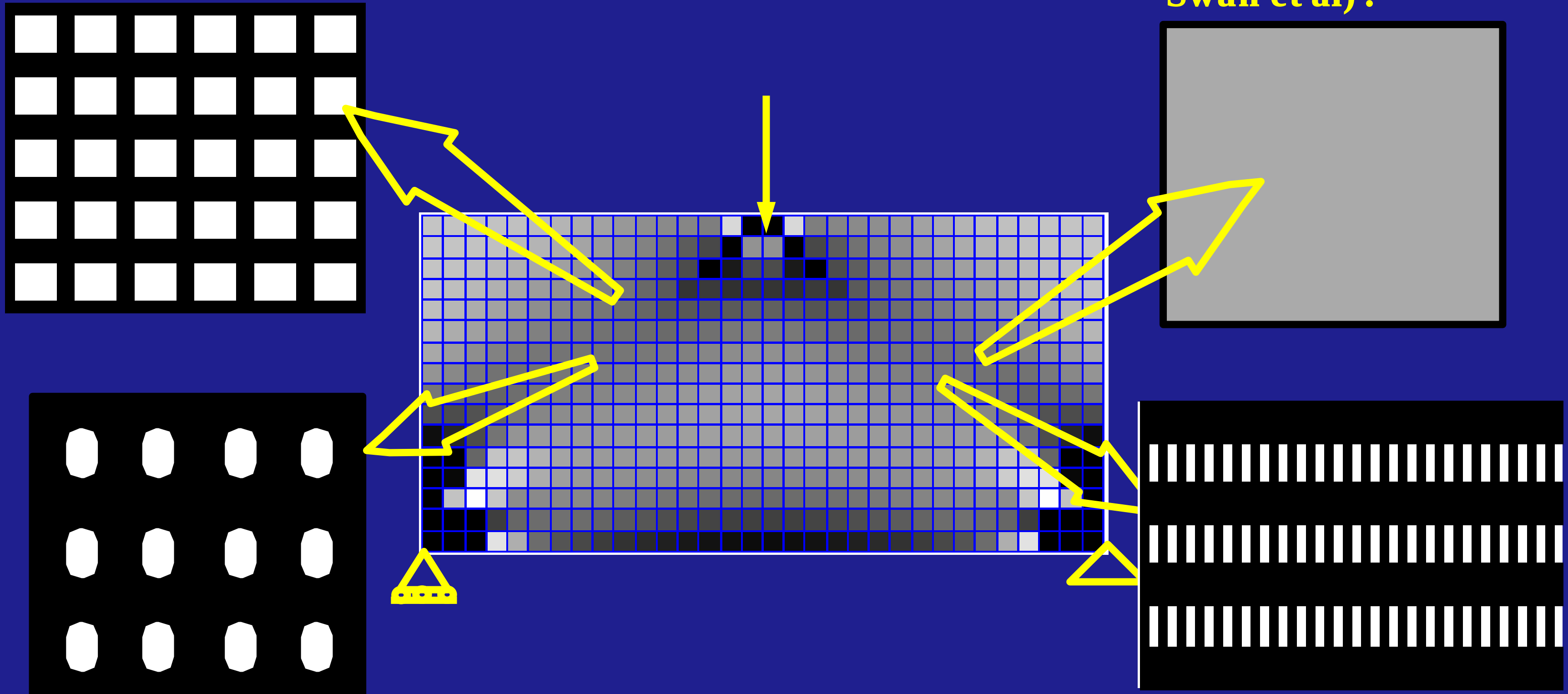
A. Brief Summary of Continuum Topology Optimization

- General material arrangements described with distributed parameters (volume fractions, micro−structure) throughout the spatial domain. For example:

$b= \{\phi_1,\phi_2, \ldots, \phi_N\}$ **the design vector**

- Structure is modeled/analyzed as a continuum. Analysis models can therefore be large and expensive.
- Wide variety of possible performance objectives/constraints
- Due to continuity and number of design variables, gradient based optimization methods are used.
- The optimization problem is typically non−convex with many local optima.

Samples of Continuum Formulations



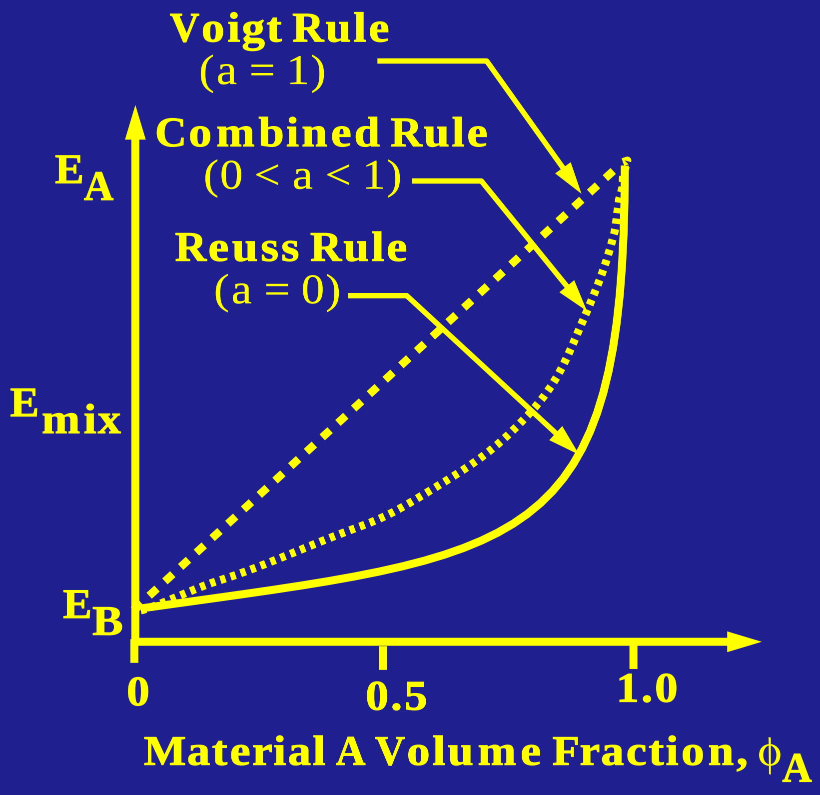
Porous solid micro-structure (Bendsoe and Kikuchi).

Microstructure-free mixture (Sigmund; Swan et al) .

Elliptical inclusion in matrix microstructure (Gea).

Rank-2 laminate microstructure (Allaire and Kohn; Jog, Haber and Bendsoe)

Treatment of "Grey" Elements Containing Material Mixtures



Stiffness Behavior

Elastic Compliance Minimization Problems

Example: For a linear, elastic structural system:

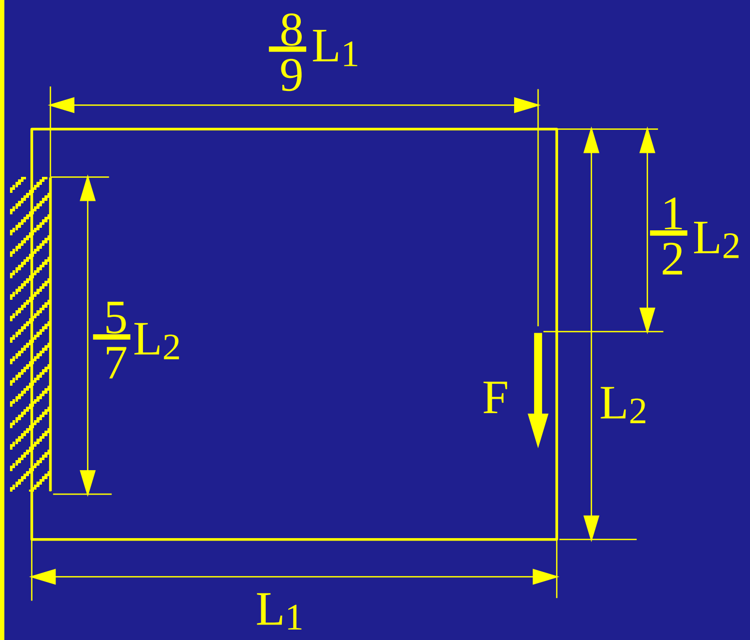
$$\min_{b} \ \Pi(b) \quad \text{subject to:} \quad \begin{array}{ll} r(b,u) & = 0; \\ \langle \phi_A \rangle - C_A & \leq 0. \end{array}$$

Optimization problem is solved using SLP.

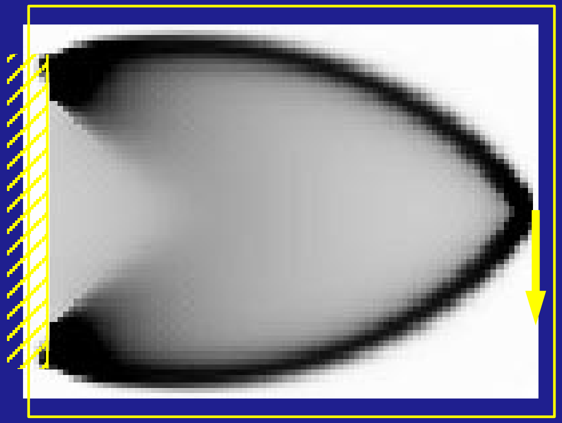
There are a wide variety of alternative elastic/inelastic problem formulations.

Characteristics of Continuum Topology Solutions

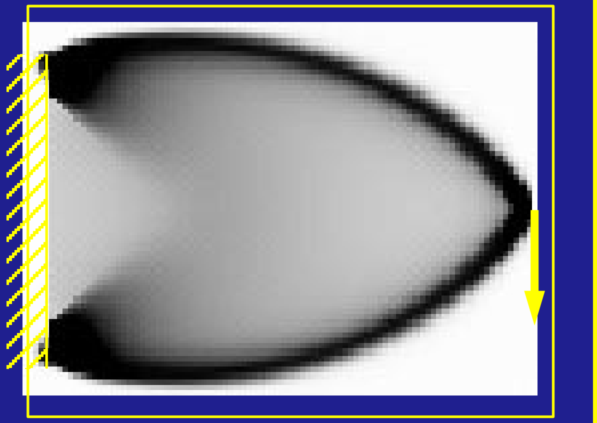
Short Cantilever Beam Design Problem



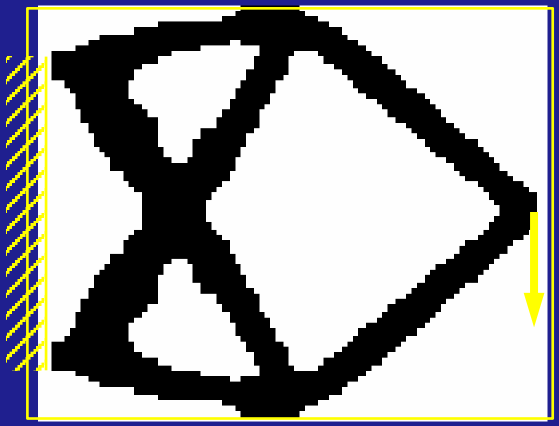
a) $L_1 = 90$; $L_2 = 70$; $F = 10^3$
$E_{solid} = 7\text{x}10^9$; $\nu = 0.333$



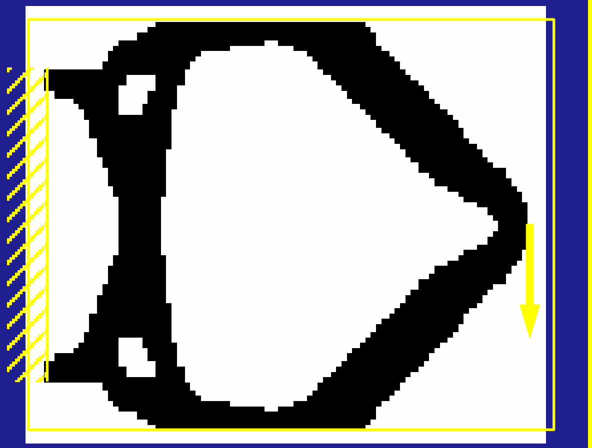
b) Voigt solution; $\Delta_m = 0.05$; $b^0 = 1.0$; $\Pi = 2.07\text{x}10^{-3}$



c) Voigt solution; $\Delta_m = 0.05$; $b^0 = 0.3$; $\Pi = 2.07\text{x}10^{-3}$



d) Reuss solution; $\Delta_m = 0.05$; $b^0 = 1.0$; $\Pi = 3.16\text{x}10^{-3}$



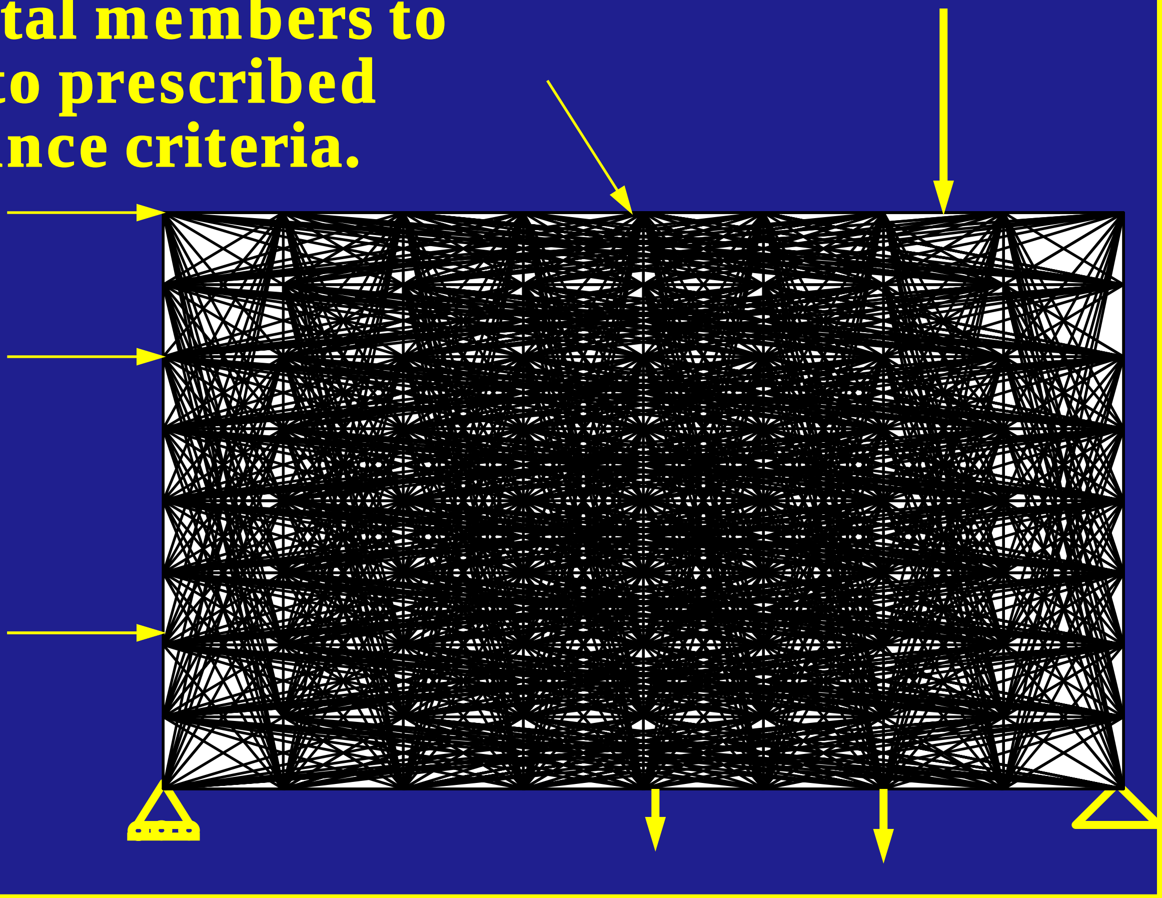
e) Reuss solution; $\Delta_m = 0.05$; $b^0 = 0.3$; $\Pi = 1.17\text{x}10^{-2}$

B. Discrete Ground−Structure Formulation

- Discretize structural domain into a finite spatial distribution of nodes.

- Connect all nodes using truss members.

- Retain only the most vital members to optimize with respect to prescribed loadings and performance criteria.



Typical Elastic Design Problem Formulation

$$\min_{b} \sum_{i} (\rho A L)_i \text{ such that:}$$

1) $r(b,u) = 0;$ **(equilibrium)**

2) $\dfrac{\Pi}{\Pi_{\text{allowable}}} - 1 \leq 0;$ **(compliance)**

3) $\dfrac{|\sigma|}{\sigma_{\text{allowable}}} - 1 \leq 0;$ **(stress constraints)**

4) $\dfrac{-P}{P_{cr}} - 1 \leq 0;$ **(local buckling constraints)**

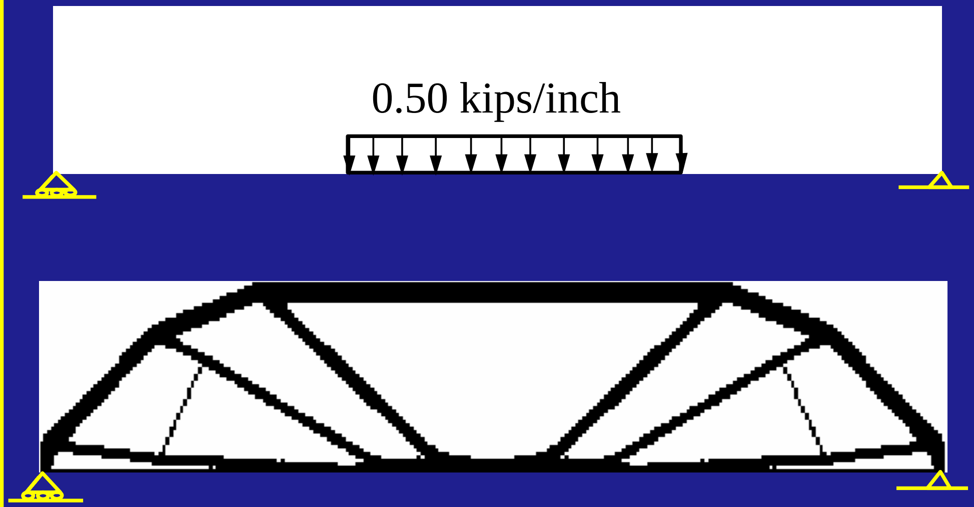
5) $\dfrac{-NEL}{NEL_{\text{allowable}}} - 1 \leq 0;$ **(member count constraint)**

- Design variables are truss member section properties.
    - Typically area, with moment of inertia;
- Problems are solved using genetic algorithms
    - Require no design gradients;
    - Can, in principle, achieve global optimum;
    - Used SAGA software (Arora and Wang, 1996);
    - Used fitness function of Kocer (1998).
- Constraints on problem size:
    - As number of truss members and discrete design variable values increases, number of design possibilities quickly -> ∞.
    - Consequently, only problems with coarse node distributions and few sectional possibilities can presently be solved.

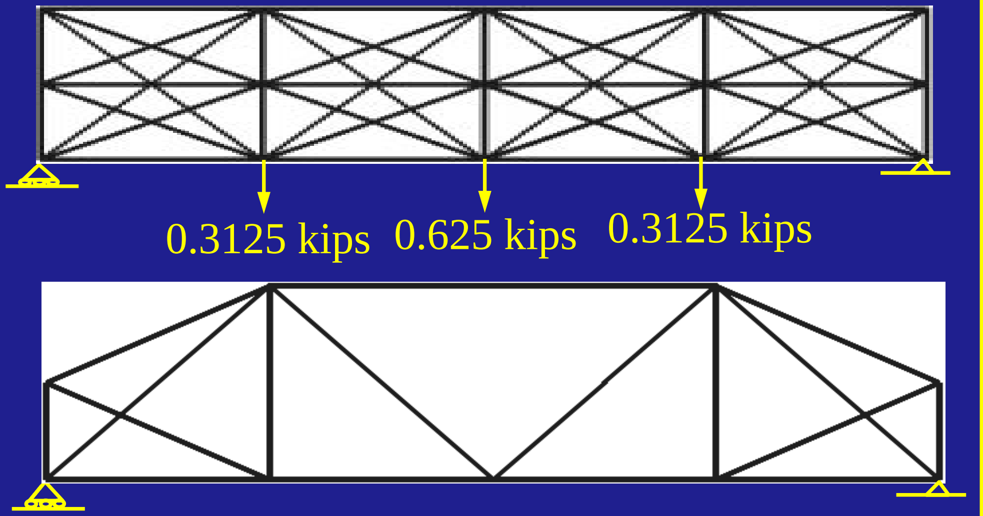
C. Comparison of Methods on a Design Problem

Simply supported, 20’ x 5’ truss.

Continuum Problem



Discrete Problem



D. Comparison of solutions:

- **Computational expense**
  - **continuum: 2 cpu−hours on SGI PowerChallenge (single analysis cost significant, but few design iterations required)**
  - **discrete : 1.5cpu−hours on HP−715 (single analysis cost trivial, but many analyses required)**
- **Design space (structural possibilities)**
  - **continuum clearly allows "many" more arrangements of members than discrete**
- **Other Considerations:**
  - **discrete allows modeling of cross−sections;**
  - **continuum designs tend to be "unrealistically heavy" due to continuum modeling.**

E. Conclusions

1) Discrete methods seem more naturally suited to sparse civil structures using beam/truss type structural members.

2) There are barriers to solving large 3D structural concept design problems with both approaches:

- continuum: analysis cost
- discrete : excessive design possibilities