On the Role of Continuum Structural Topology Optimization in Concept Design of Civil Structures

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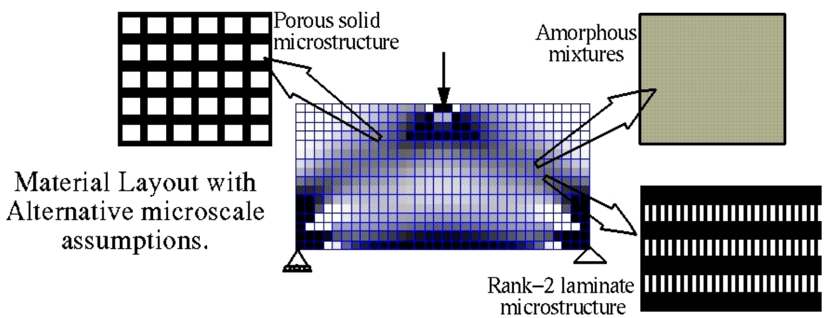
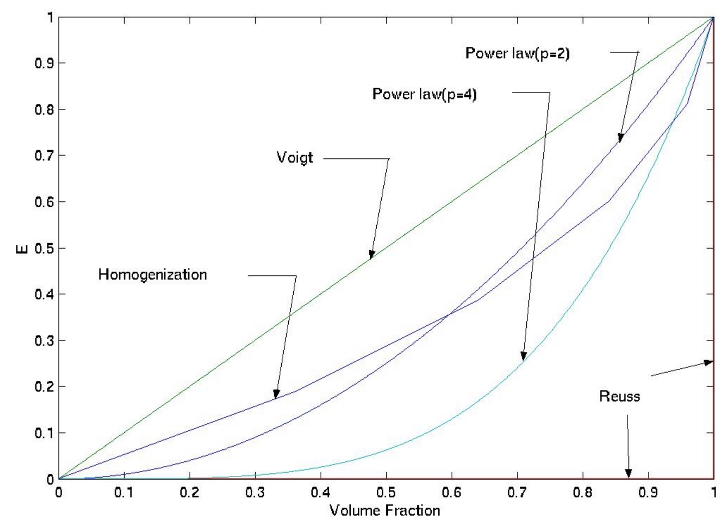
Brief Summary of Continuum Structural Topology Optimization

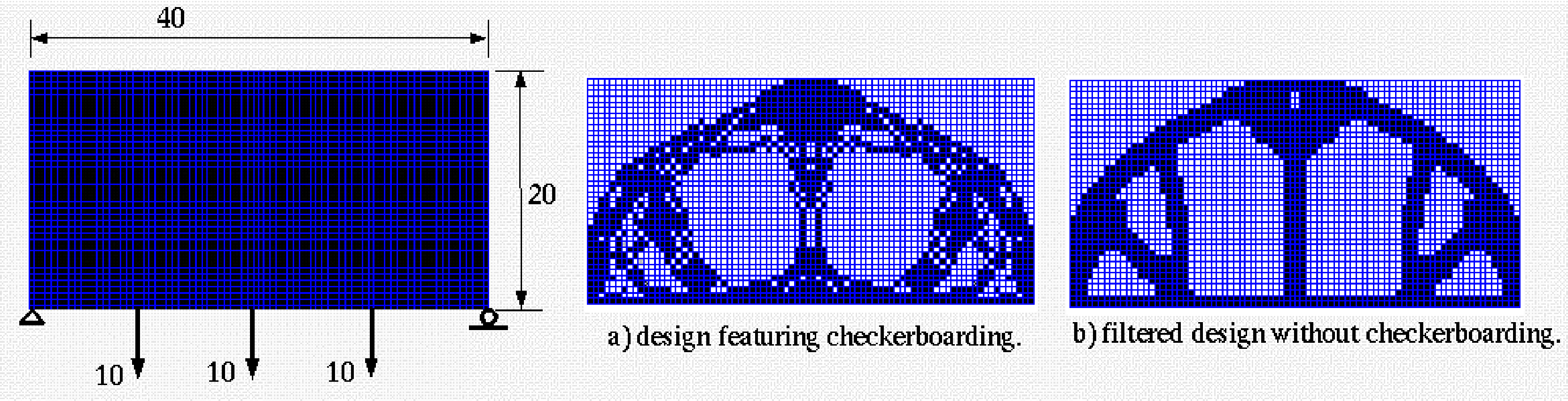
- Structural domain is discretized into a mesh of volume/area elements.
- A solid volume-fraction design variable is associated with each element.
- A "mixture-model" is used to designate element-level material stiffness as a function of solid volume fraction.
- An optimization problem is formulated and solved to maximize structural performance, subject to material usage constraints.





Characteristics of continuum structural topology optimization as applied to civil structures

- The element-based numerical formulation has instabilities:
  - results in "checkerboarding" designs.
- As modeled, continuum structures are unrealistically "heavy".
  - lack the sparsity of civil structures
  - continuum joints transmit moments
  - cannot capture potential buckling behaviours
- The problem admits a large number of locally optimal design solutions.
- Here, we attempt to address the first two problems.



a) design featuring checkerboarding.

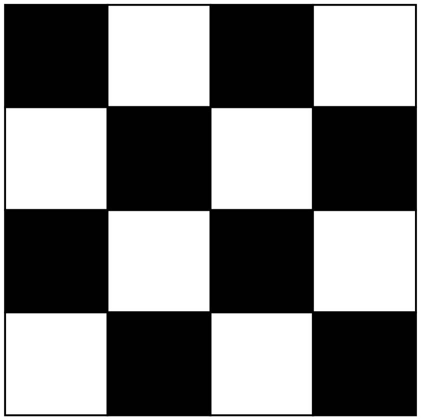
b) filtered design without checkerboarding.

Element-based design variables:

•solid volume-fraction design field can be discontinuous across element boundaries.

• this can lead to the phenomenon of “checkerboarding” design solutions.

• checkerboarding designs can be eliminated via a number of ad-hoc filtering techniques.

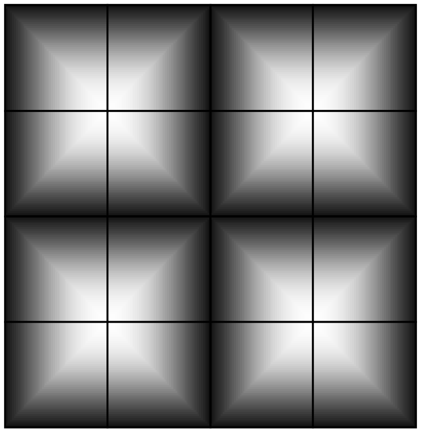


Node-based design variables:

• forces continuity of solid volume-fraction design variables across element boundaries.

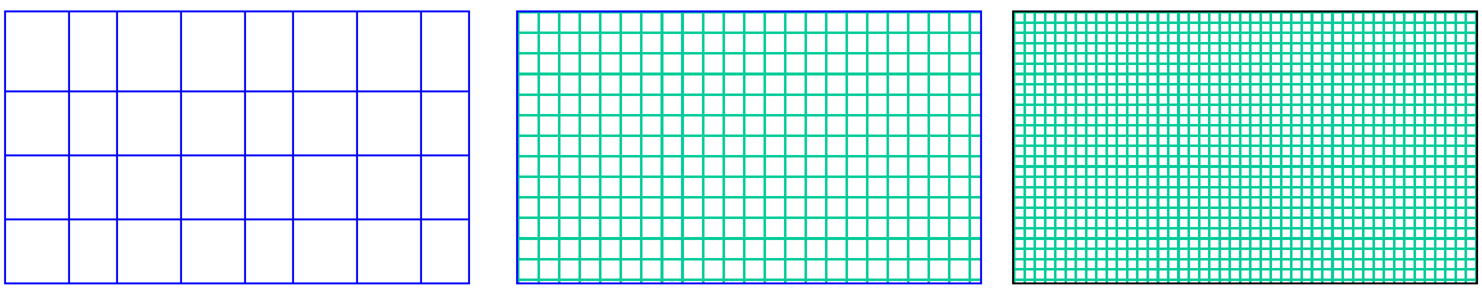
• does not admit “checkerboarding” design solutions.

• requires no spatial filtering techniques.



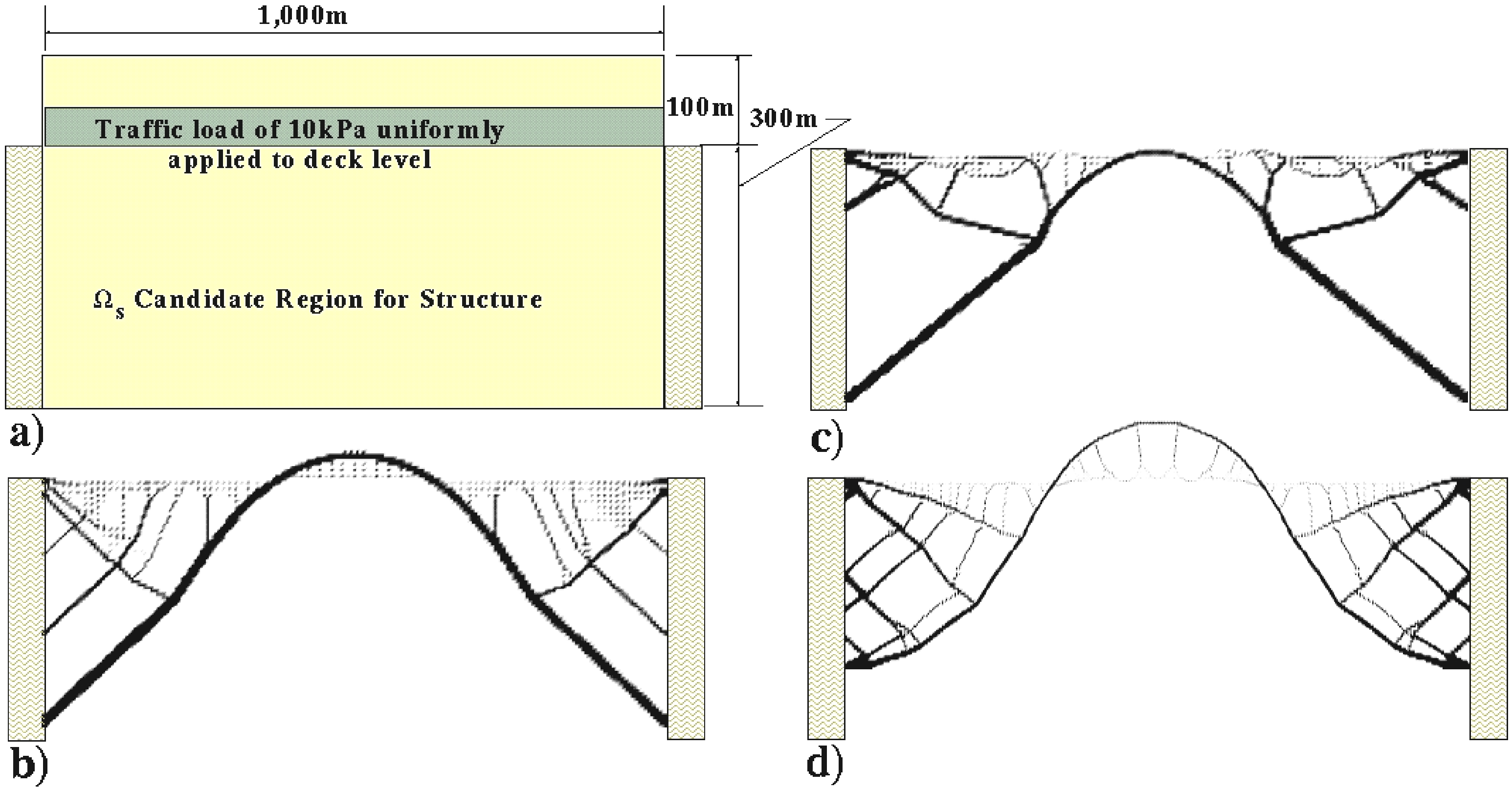
Issue of Structural SPARSITY

- Typically, large scale civil structures (as bridges) are sparse
    - occupy only a small fraction of the structure's envelope volume.
- In design optimization, structural models must capture the characteristic sparsity to yield realistic performance.
- When modeling sparse structures with continuum models, very fine meshing is required.
    - This adds to the computational expense of the analysis and design process.



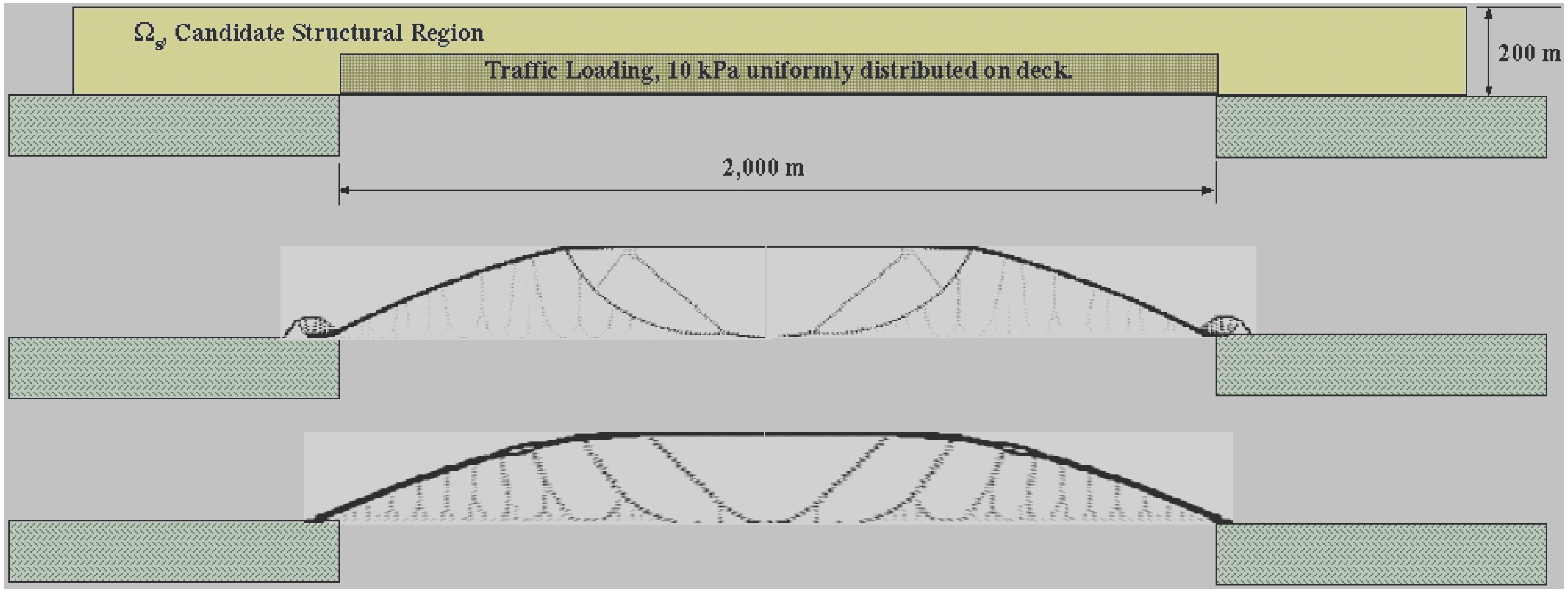
Sparse, Canyon Bridge Concept Design Problem:

- span = 1000m; height = 400m;
- design traffic loading (AASHTO) on deck is 10kPa
- design can occupy only 5% of gross volume 400,000 m^3/m
- bridge weighs 1560 MN/m; design load is only 10 MN/m
- 16000 elements, 8000 design variables used.



Sparse, Long Span Concept Design Problem:

- span = 2000m; height = 200m;
- design traffic loading (AASHTO) on deck is 10kPa
- design can occupy only 5% of gross volume 400,000 m^3/m
- bridge weighs 2340 MN/m; design load is only 20 MN/m
- 32000 elements, 16000 design variables used.



Observation: Designs minimizing linear elastic compliance do not result in result in tension/suspension designs as would be expected.

Accounting for structural buckling behaviours

- Structure is modeled as hyper-elastic undergoing finite deformation effects

- Constitutive law

$$U(J)=\frac{1}{2}K[\frac{1}{2}(J^2-1)-\ln(J)] \qquad \bar{W}=\frac{1}{2}\mu[tr(\dot{\boldsymbol{\theta}})-3]. \qquad \tau = JU^{'}(J)\mathbf{1}+2dev\frac{\partial \bar{W}}{\partial \boldsymbol{\theta}}.$$

- Variational formulation

$$\int_{\Omega_S} \tau_{ij}\delta\varepsilon_{ij}d\Omega_S = \int_{\Omega_S} \rho_o\gamma_j\delta u_j d\Omega_S + \int_{\Gamma_h} h_j\delta u_j d\Gamma_h \qquad d(\delta W^{\text{int}}) = \int_{\Omega_S} \delta\varepsilon_{ij}dL_v(\tau_{ij})d\Omega_S + \int_{\Omega_S} \delta\varepsilon_{ji}\tau_{im}d\varepsilon_{jm}d\Omega_S,$$

- Discrete Equilibrium

$$\boldsymbol{r}_{n+1}^A = (\boldsymbol{f}^{\text{int}})_{n+1}^A - (\boldsymbol{f}^{ext})_{n+1}^A = 0 \qquad (\boldsymbol{f}^{\text{int}})_{n+1}^A = \int_{\Omega_S} (\boldsymbol{B}^A)^T : \boldsymbol{\tau}_{n+1}d\Omega_S \qquad (\boldsymbol{f}^{ext})_{n+1}^A = \int_{\Omega_S} \rho_o N^A\gamma_{n+1}d\Omega_S + \int_{\Gamma_h} N^A\boldsymbol{h}_{n+1}d\Gamma_h.$$

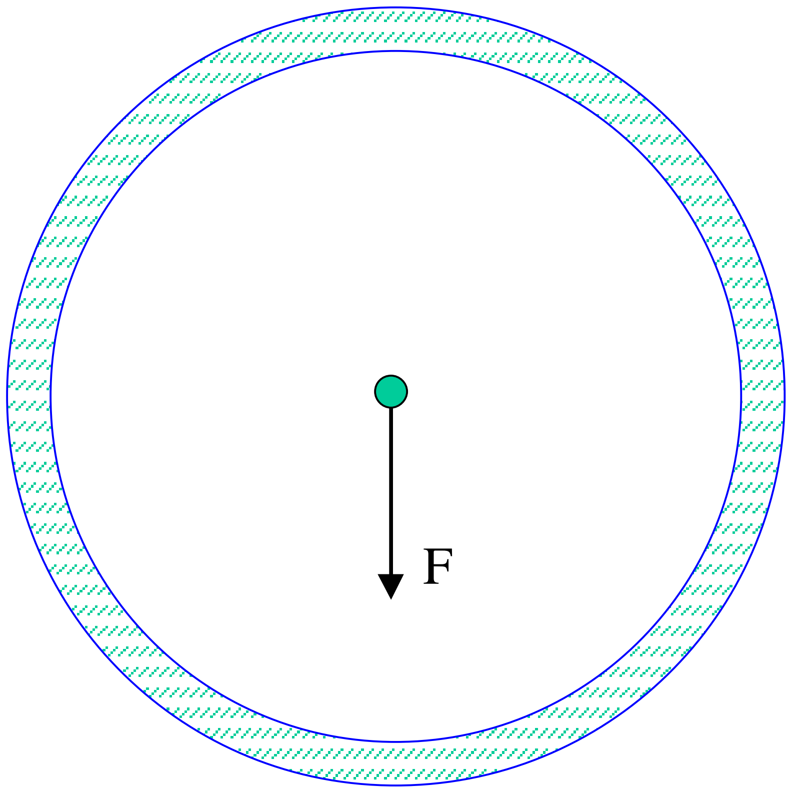
$$\boldsymbol{K}_{il}^{AB} = \int_{\Omega_s} B_{ji}^A c_{jk} B_{kl}^B d\Omega_s + \int_{\Omega_s} N_j^A\tau_{jk}N_k^B\delta_{il}d\Omega_s,$$

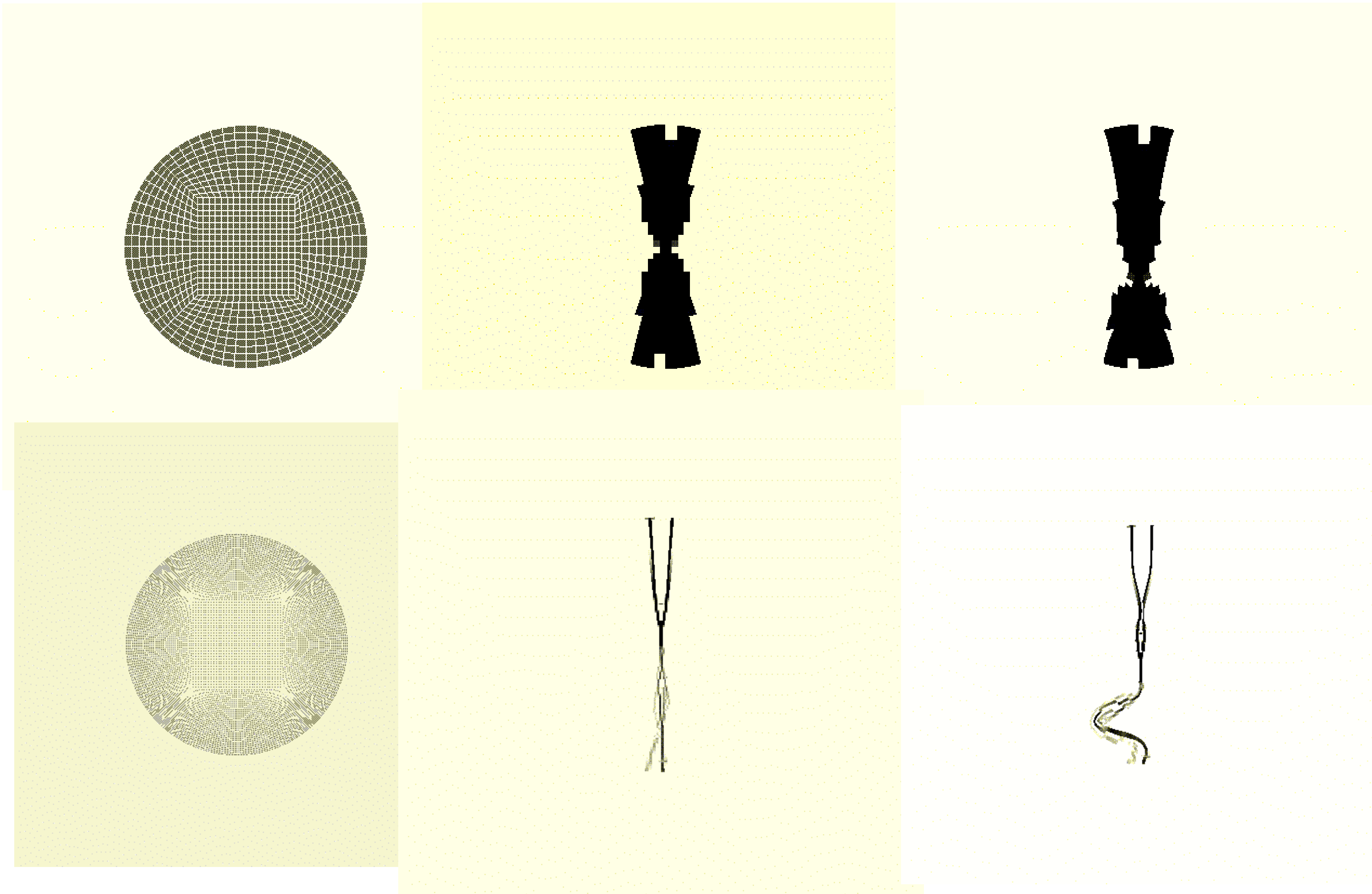
- Objective function and design derivative

$$f_E(\boldsymbol{u},\boldsymbol{b}) = \int_{\Omega_s} E(\boldsymbol{F})d\Omega_s \qquad \frac{\partial f_E}{\boldsymbol{db}} = \int_{\Omega_s} \frac{\partial E}{\partial \boldsymbol{b}}d\Omega_s + \boldsymbol{u}^a\int_{\Omega_s} \boldsymbol{B}^T.\frac{\partial \boldsymbol{\tau}}{\partial \boldsymbol{b}}d\Omega_s \qquad \boldsymbol{K}(\boldsymbol{u},\boldsymbol{b}).\boldsymbol{u}^a = -\int_{\Omega_s} \boldsymbol{B}^T.\boldsymbol{\tau}d\Omega_s.$$

Test Design Problem for Buckling Detection:

- Design optimum sparse, elastic structure in the circular domain to carry the design load back to fixed, rigid walls.





Summary and Conclusions:

• For continuum structural topology optimization methods to be useful in concept design of large-scale, civil structures, they must be able to detect potential buckling instabilities.

• Requirements:

  • modeling of sparse structures at high mesh refinement.

  • solution of structural analysis problems with geometric nonlinearity.

• Pure gradient-based design methods encounter difficulty in obtaining optimal pure-tension structures.

• Further development is thus needed.

Question: Do continuum structural topology optimization methods have a role to play in concept design of large-scale civil structures?

Answer: Not yet.