Poroelastic Modeling of Fluid Flow at the Haversian and Lacunar Scales

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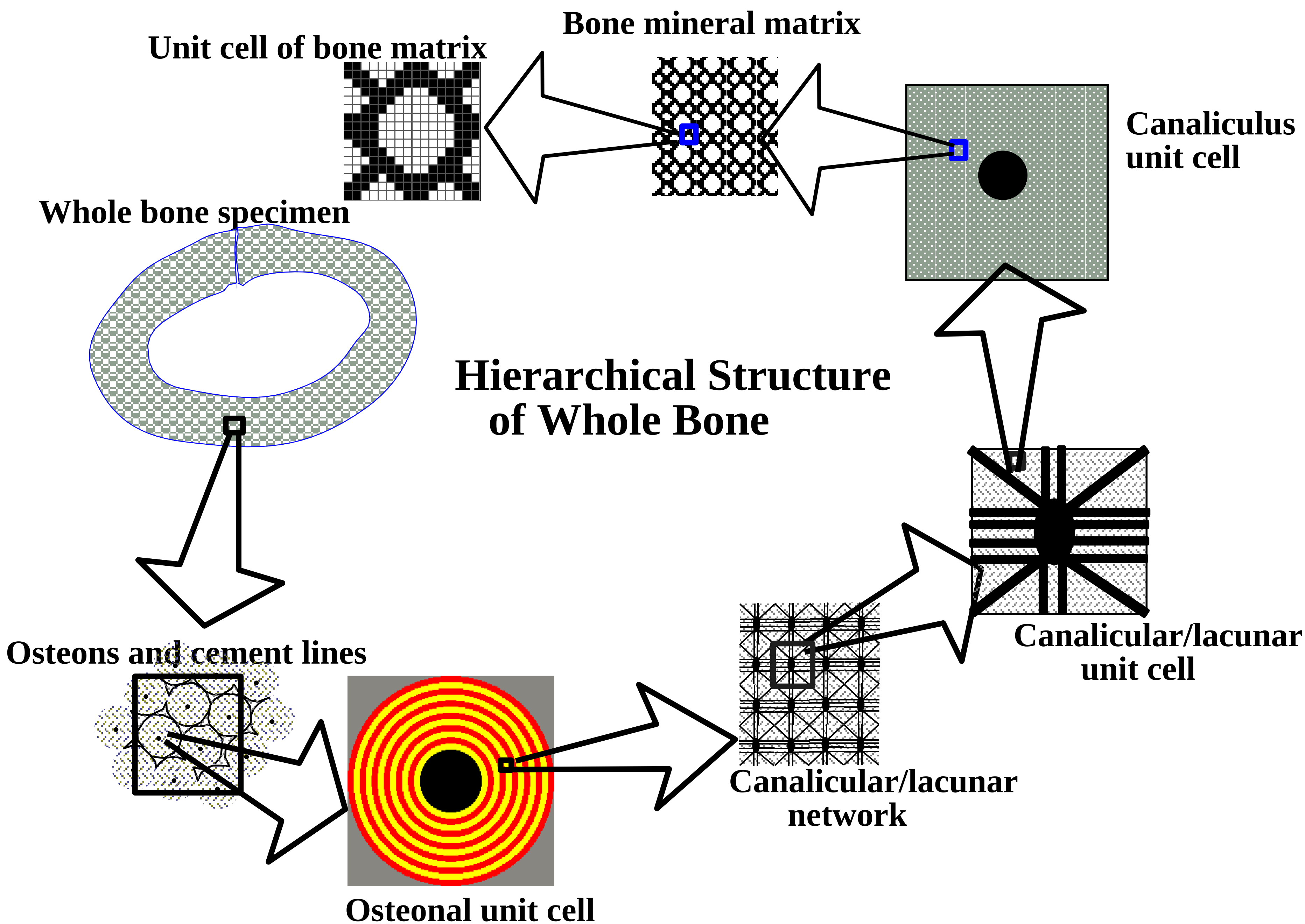


Bone mineral matrix
Unit cell of bone matrix
Canaliculus unit cell
Whole bone specimen
Hierarchical Structure of Whole Bone
Osteons and cement lines
Canalicular/lacunar unit cell
Canalicular/lacunar network
Osteonal unit cell

Orthotropic Poroelastic Constitutive Model for Cortical Bone on Multiple Length Scales (Biot, 1956, 1957, 1962)

$$
\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \\ p_f \end{bmatrix}
=
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 & C_{17} \\
C_{21} & C_{22} & C_{23} & 0 & 0 & 0 & C_{27} \\
C_{31} & C_{32} & C_{33} & 0 & 0 & 0 & C_{37} \\
0 & 0 & 0 & C_{44} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66} & 0 \\
C_{71} & C_{72} & C_{73} & 0 & 0 & 0 & C_{77}
\end{bmatrix}
\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \\ \zeta \end{bmatrix}
$$

σ values denote total stresses;
ε values denote matrix strains;
p_f denotes fluid pressure;
ζ denotes change of fluid content;

*All constants in this poro−elastic model can be determined from micromechanical analysis.

The model predicts larger fluid pressures under transverse loadings.

Bone Permeabilities on Different Length Scales

Haversian/Osteonal Scale
[Rouhana *et al* , 1980]

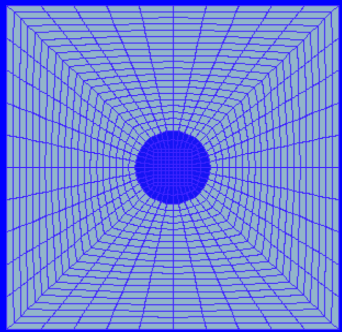
$k_{longitudinal} = 5 * 10^{-13}\ m^2$
$k_{transverse} = 5 * 10^{-14}\ m^2$

Lacunar/Canalicular Scale
[Cowin *et al*, 1998, 1999]

$k_{longitudinal} = 5 * 10^{-17}\ m^2$
$k_{transverse} = 5 * 10^{-19}\ m^2$

UNIT CELL MODELING ASSUMPTIONS

- Bone matrix is homogeneous, isotropic, elastic ($E = 12$ GPa, $\nu = 0.38$). (Lamellar structure neglected)
- Fluid is elastic, with no shear viscosity on microscale ($K = 2.1$ GPa).
- The bone is fully saturated.
- Canal matrix in unit cell denotes:
  - Haversian canal at osteonal scale;
  - canaliculus at lacunar scale;



4% fluid−filled porosity

UNIT CELL MODELLING RESULTS

- All parameters in the poroelastic model are computed.
- Physical observation:
  - Loading along longitudinal canal axis generates "small" fluid pressures.
  - Loading transversely to canal axis generates larger fluid pressures.

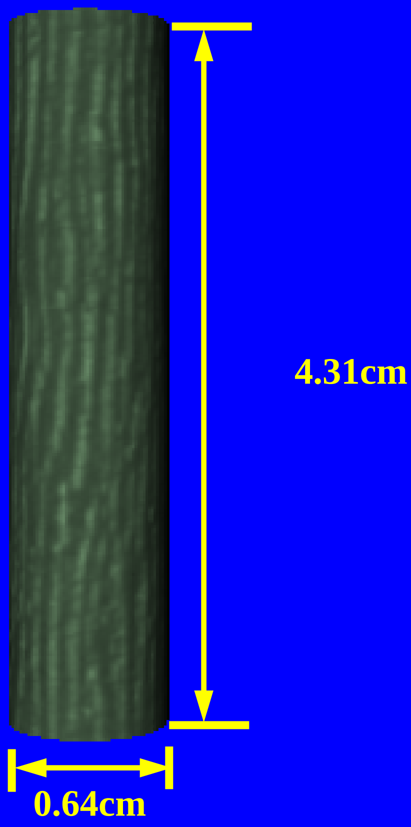
CORTICAL BONE SPECIMEN

Experiments:

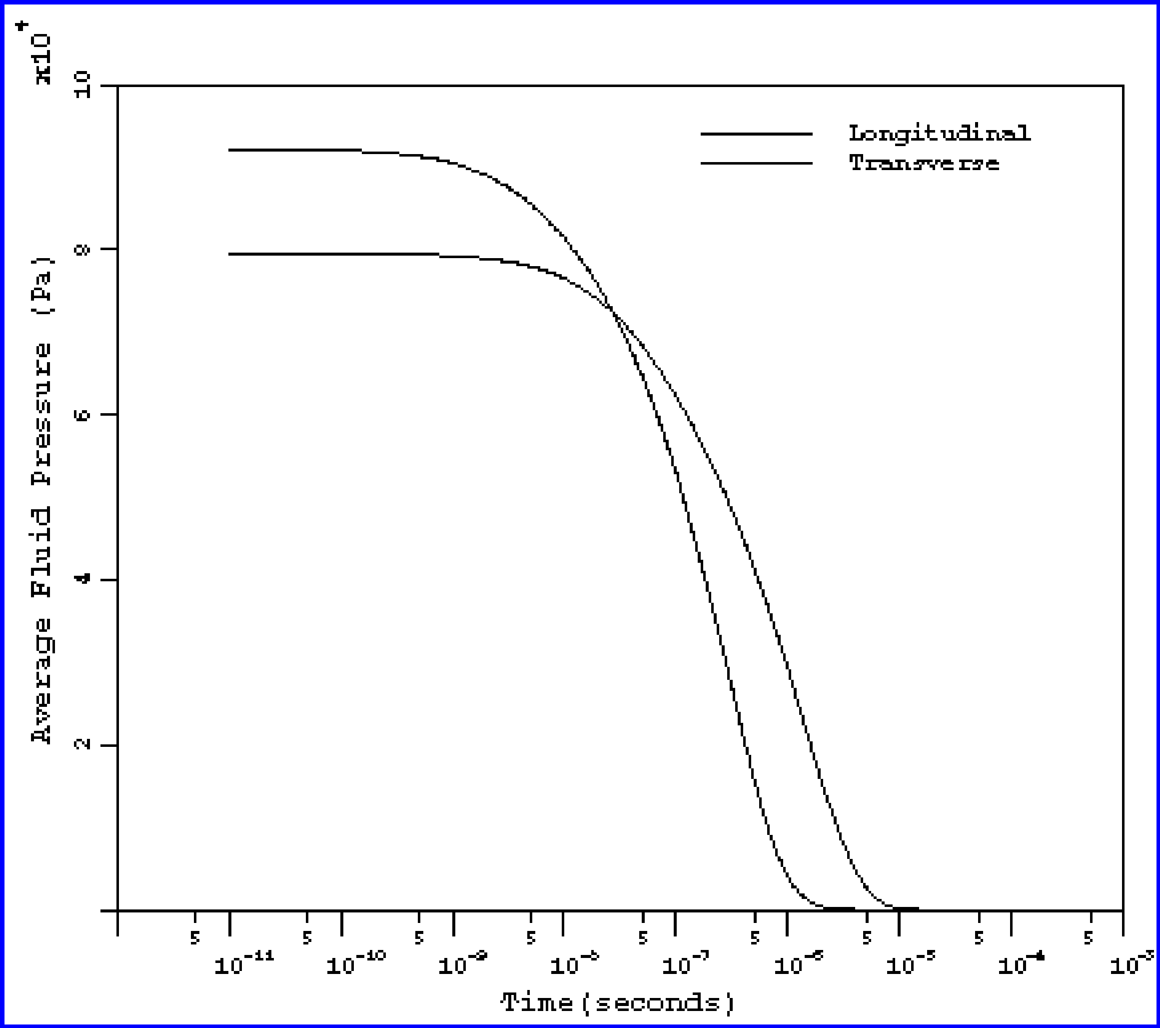
- Dynamic bending/torsion excitation.
  - Measure viscoelastic damping characteristic tan(δ).
- Air Dry and Saturated

Analysis:

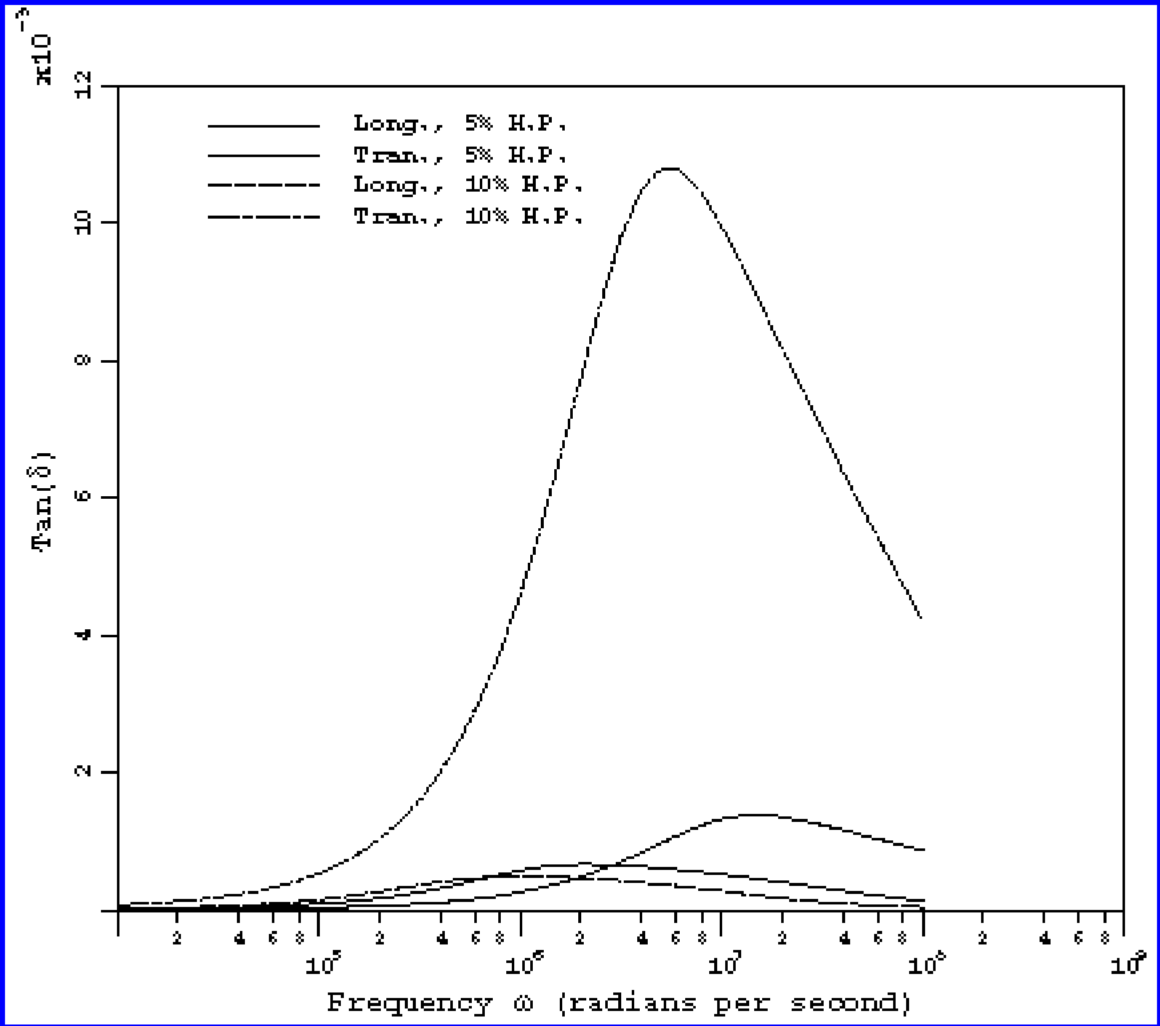
- Pressure relaxation under step−loading.
  - Compute viscoelastic damping characteristic tan(δ).
- Fully Saturated Specimen



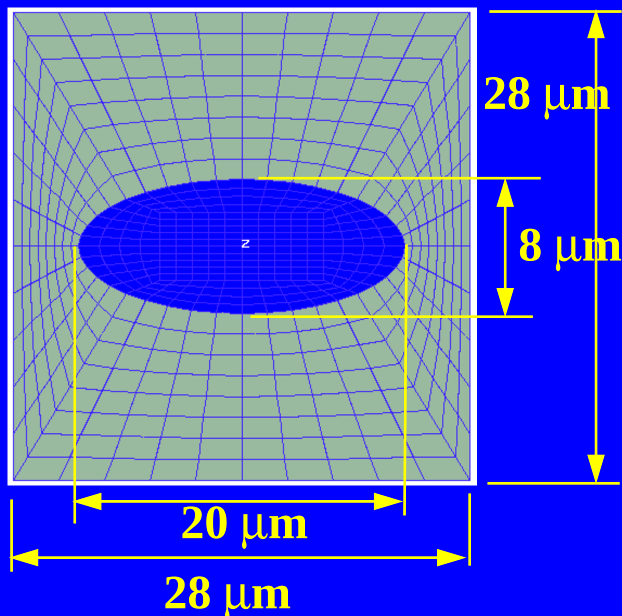
Computed Haversian Pressure Relaxation Behaviors



Computed Viscoelastic tan(δ) Behaviors Associated with Fluid Flow in Haversian System.



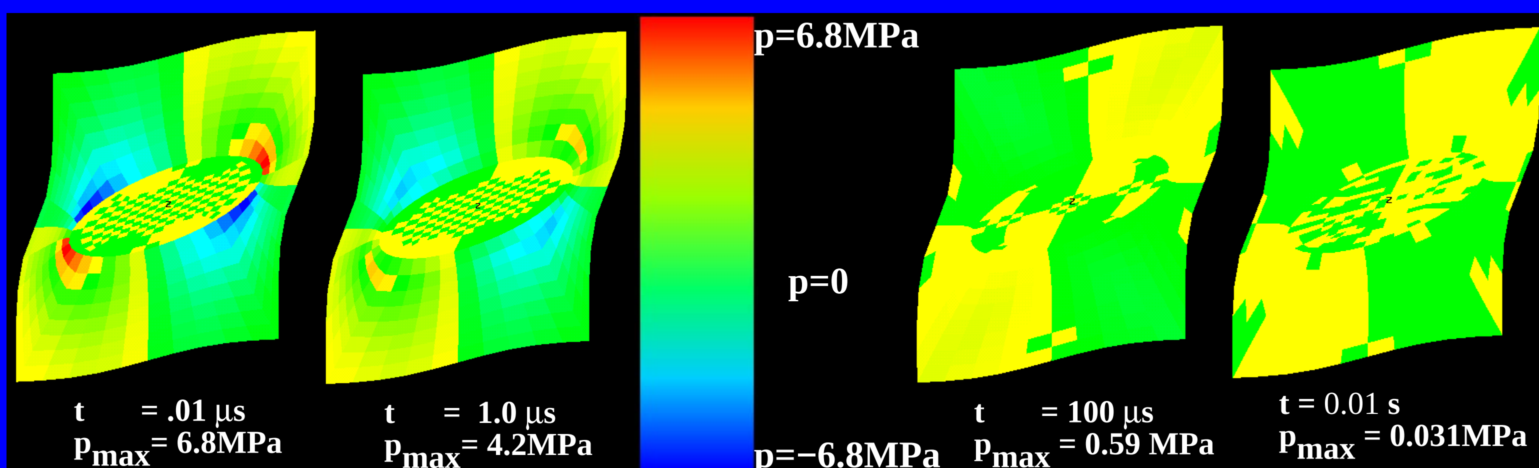
Shear−Induced Fluid Flow at Lacunar/Canalicular Scale



Applied Loading is 1% Shear Strain

Bone matrix is poro−elastic, with anisotropy due to canaliculi.

Induced fluid pressures in the canaliculi dissipate on the order microseconds.



Space/Time Fluid Pressure Distributions in Lacunar Unit Cell

FINDINGS:

- The Haversian and Volksman canals function as freely draining conduits under mechanical excitation applied well below 1 MHz.
- On the lacunar scale, load−induced fluid pressures in canaliculi relax quickly [O(1 − 100μs)] into lacunae.
- Fluid pressure relaxation frequencies on both the whole−bone and the lacunar length scales are on the order of 1−10 MHz. These are much larger than what are thought to the physiologically meaningful frequencies (.1 Hz − 1 kHz) .
- Our extensive experimental measurements of viscoelastic energy dissipation in cortical bone show no evidence of a Debye peak associated with pressure−driven fluid flow.