CONTINUUM STRUCTURAL TOPOLOGY OPTIMIZATION

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CEE-SMM Graduate Seminar
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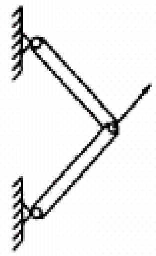
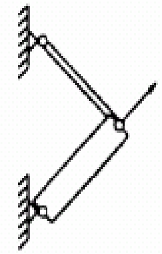
Introduction

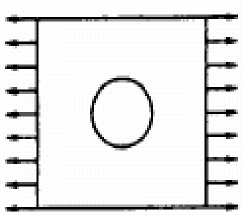
Structural Optimization

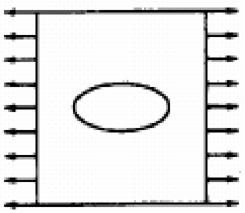
. Size Optimization



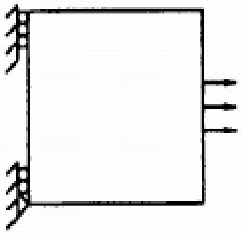


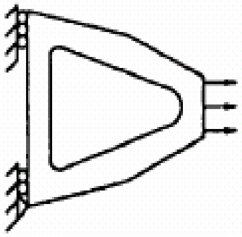
. Shape Optimization



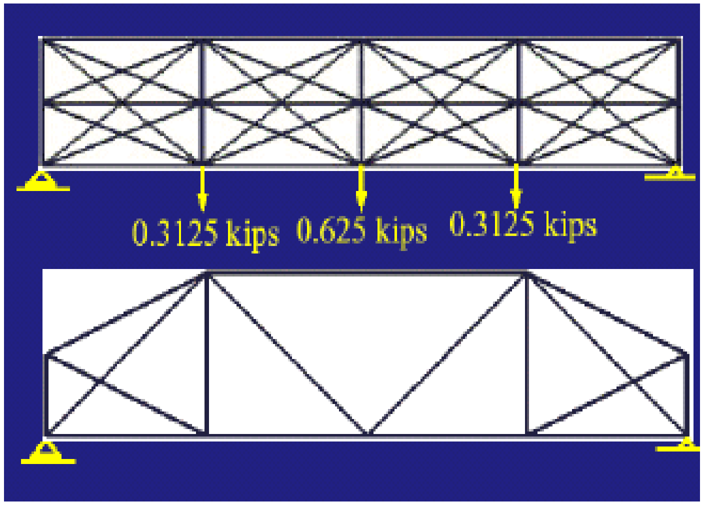


. Topology Optimization

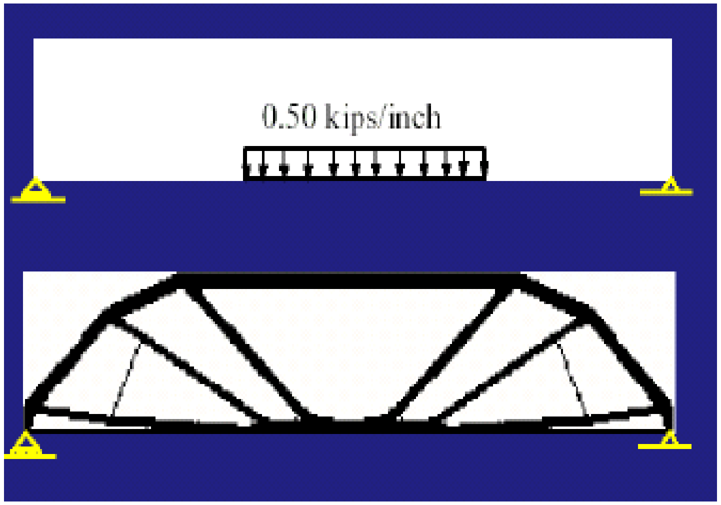




Alternative Approaches to Structural Topology Optimization



Discrete structural model of beam/truss elements

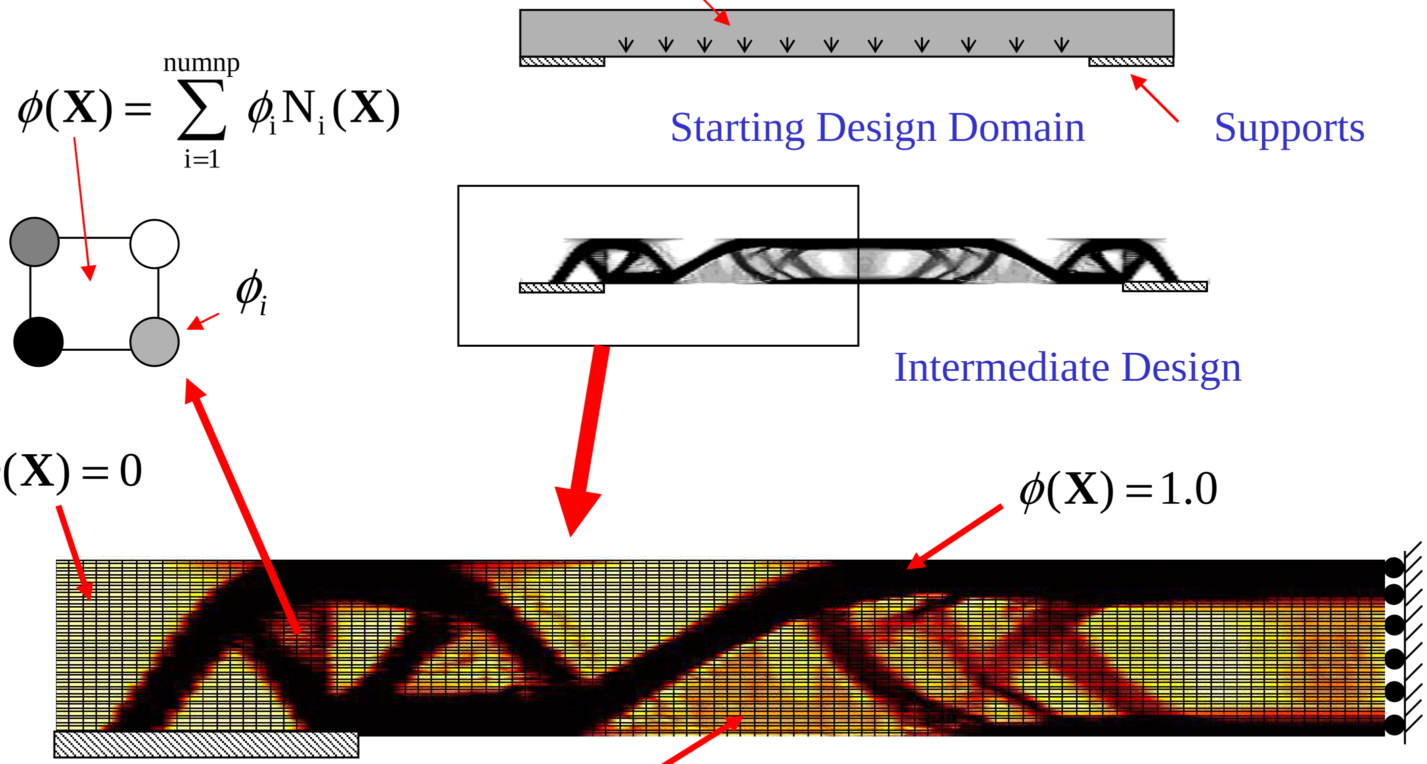


Continuum structural model

OUTLINE:

- PRELIMINARY IDEAS
  - The analysis/design framework
    - How to capture all possible structural forms?
  - Model sparsity issue
  - Problem size reduction
- APPLICATIONS
  - Design for minimal compliance
  - Design for stability

The Analysis and Design Framework



$$\phi(\mathbf{X}) = \sum_{i=1}^{\text{numnp}} \phi_i N_i(\mathbf{X})$$

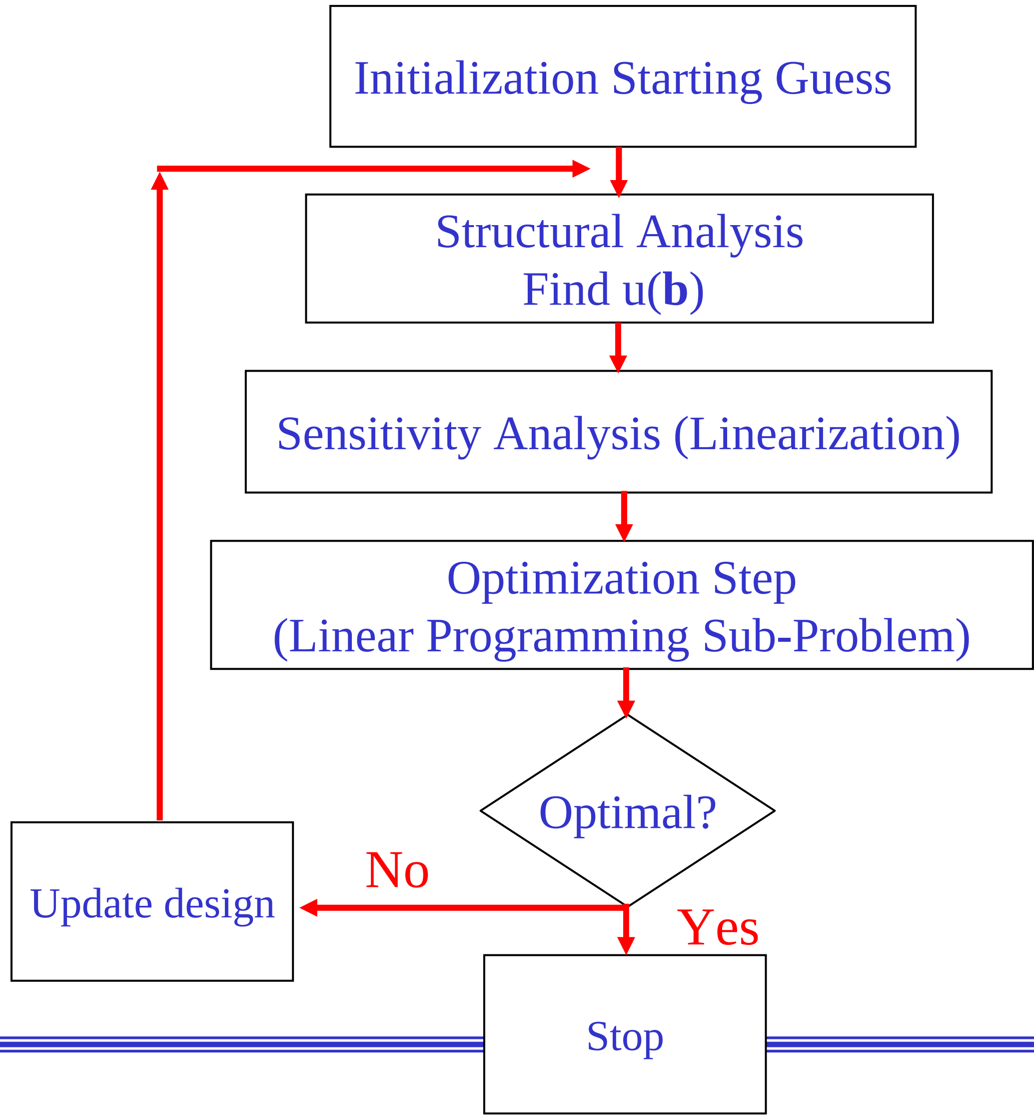
ϕ_i

$\phi(\mathbf{X}) = 0$

$\phi(\mathbf{X}) = 1.0$

$0 \leq \phi(\mathbf{X}) \leq 1.0 \qquad E(\mathbf{X}) = E(\phi(\mathbf{X}))$ [mixing rule]

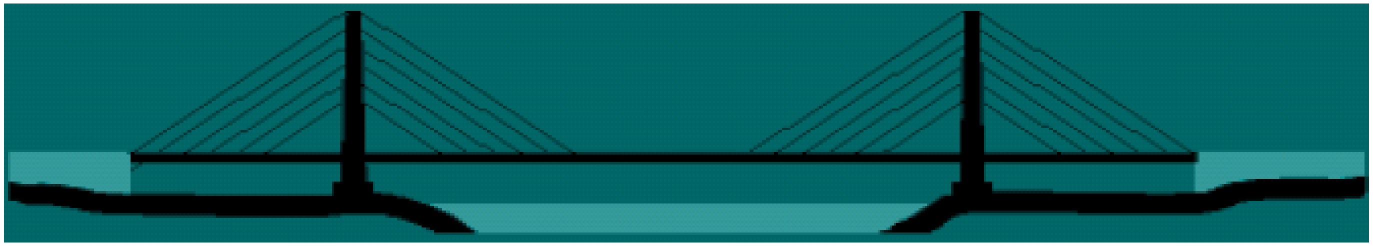
Gradient-Based Optimization Algorithm



Sparsity of Long-Span Bridges

Sunshine Skyway bridge cable-stayed bridge in Tampa, Florida

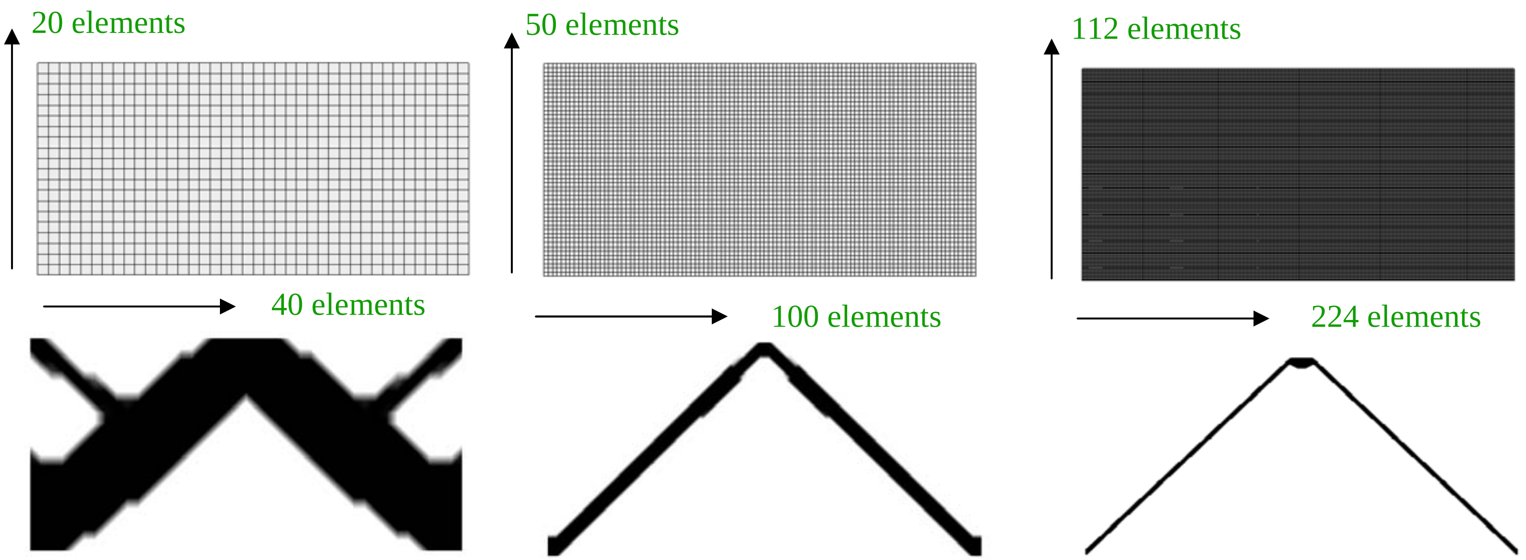
Akashi Bridge suspension bridge In Japan



Most long-span bridges occupy < 1% of their envelope volume.

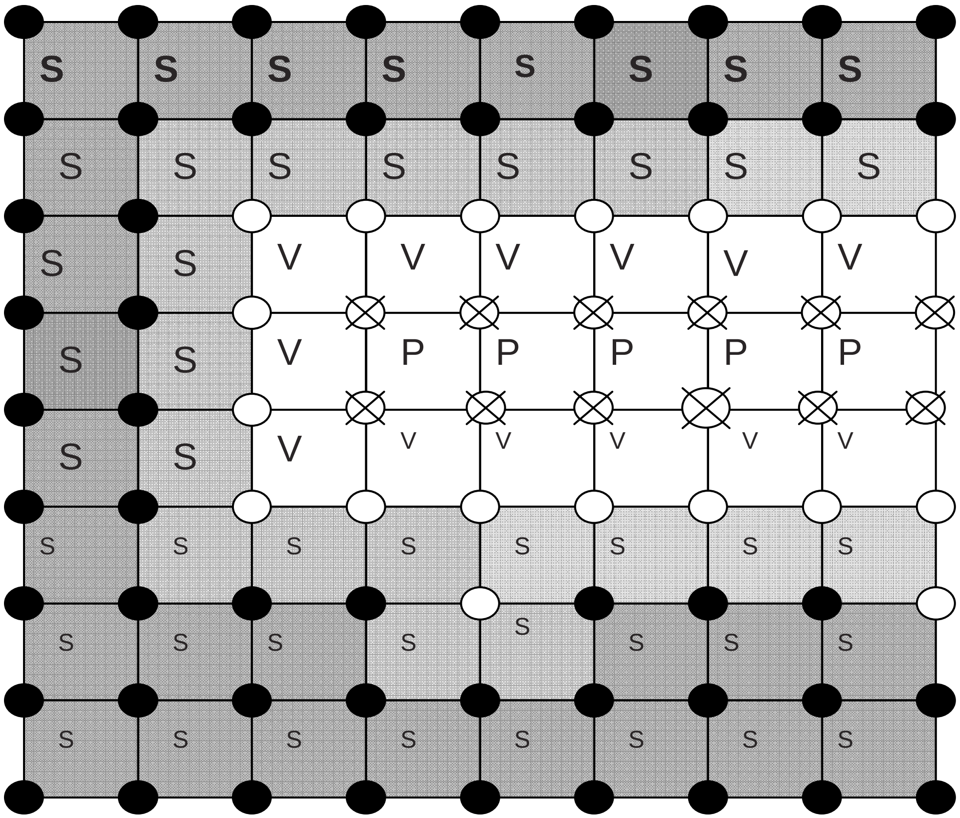
Capturing SPARSITY in Continuum Topology Optimization

- Fixed-mesh model of full envelope volume;
  - must capture the form of the structure with realistic sparsity
  - must capture mechanical performance of the structure
- Fine meshes are required;
- Implies large computational expense;



50% Material constraint usage | **10%** Material constrain usage | **3%** Material constraint usage

Analysis Problem Size Reduction



Nodes having vanishing design variable values are denoted with open circles; filled circles denote nodes associated with nonzero design values; nodes represented by open circles with X's are "prime" nodes whose degrees of freedom are restrained in the size reduction method. Elements designated with "S" are at least partially solid and those with "V" are devoid of material. Those designated with "P" are prime and need not be considered during structural analysis since all of their degrees of freedom are restrained.

Strategy for Computational Efficiency:

1. Start with coarse structural model
2. Find associated optimal arrangement of material;
3. Construct a finer structural model;
4. Map the current design onto the finer structural model;
5. Reduce allowable material usage
6. Return to step #2.

- Structure modeled as linearly elastic system
  - Stability analysis performed via linearized buckling analysis

- Linear elastic problem: $\mathbf{K}_L \cdot \mathbf{u} = \mathbf{f}^{ext}$ Compliance: $\Pi = \frac{1}{2}\mathbf{f}^{ext} \cdot \mathbf{u} = \frac{1}{2}\mathbf{u} \cdot \mathbf{K}_L \cdot \mathbf{u}$
- Eigenvalue problem: $K_L(b)\psi + \lambda G(u,b)\psi = 0$ $\lambda = -\frac{\psi^T K_L \psi}{\psi^T G \psi}$
- Objective function: $L(u,b) = \frac{1}{\min(\lambda)}$
- Design sensitivity analysis:

$$\frac{dL}{db} = -\psi^T\left(\frac{\partial G}{\partial b} + \frac{1}{\lambda}\frac{\partial K_L}{\partial b}\right)\psi + (u^a)^T\left(\frac{\partial K_L}{\partial b}.u - \frac{\partial f^{ext}}{\partial b}\right)$$

$$K_L u^a = \psi^T \frac{\partial G}{\partial u}\psi$$

Design Constraints

- Bounds on individual design variables

$$0 \le \phi_J \le 1 \quad J = 1,2,\ldots,\mathrm{NUMNP}$$

- Material usage constraint

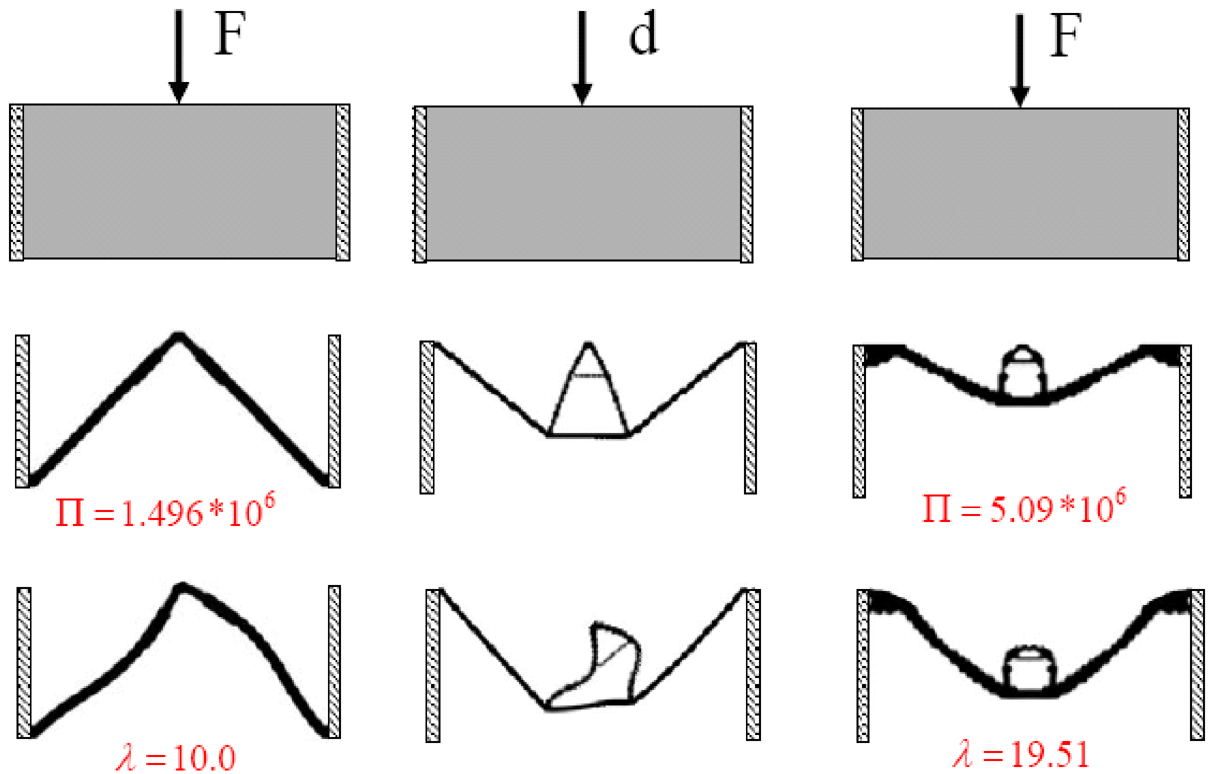
$$\frac{\int_{\Omega} \phi(\mathbf{X})\, d\Omega}{\int_{\Omega} d\Omega} \le C$$

Fixed-End Beam Problem, Design for Stability

Design domain with 10% material constraint.

Resulting topology

First buckling mode.

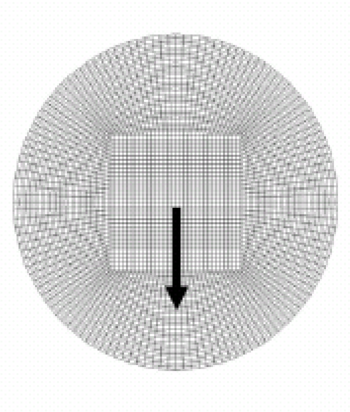


Minimizing the general compliance.

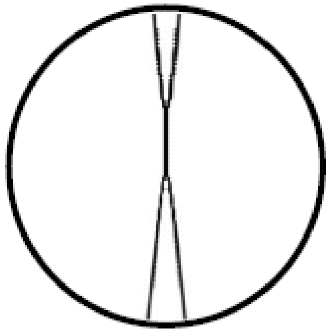
Maximizing the min. critical buckling load, nonlinear formulation.

Maximizing the min. critical buckling load, linearized buckling.

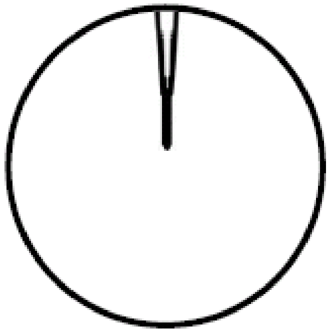
Circle Problem



Design domain
With material
2.5% Constraint

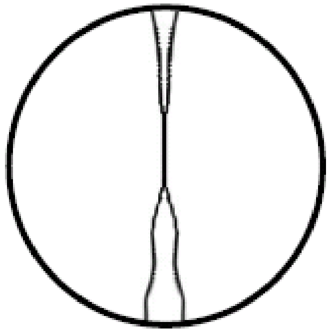


$\Pi = 4.09*10^5$



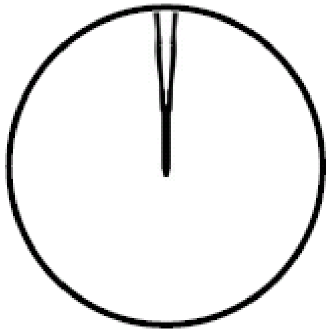
$\Pi = 2.66*10^5$

Undeformed configuration



$\lambda = 1.57*10^9$

Minimizing structural Compliance



$\lambda = 3.28*10^9$

Maximizing the min. critical buckling load, linearized buckling.

Deformed configuration

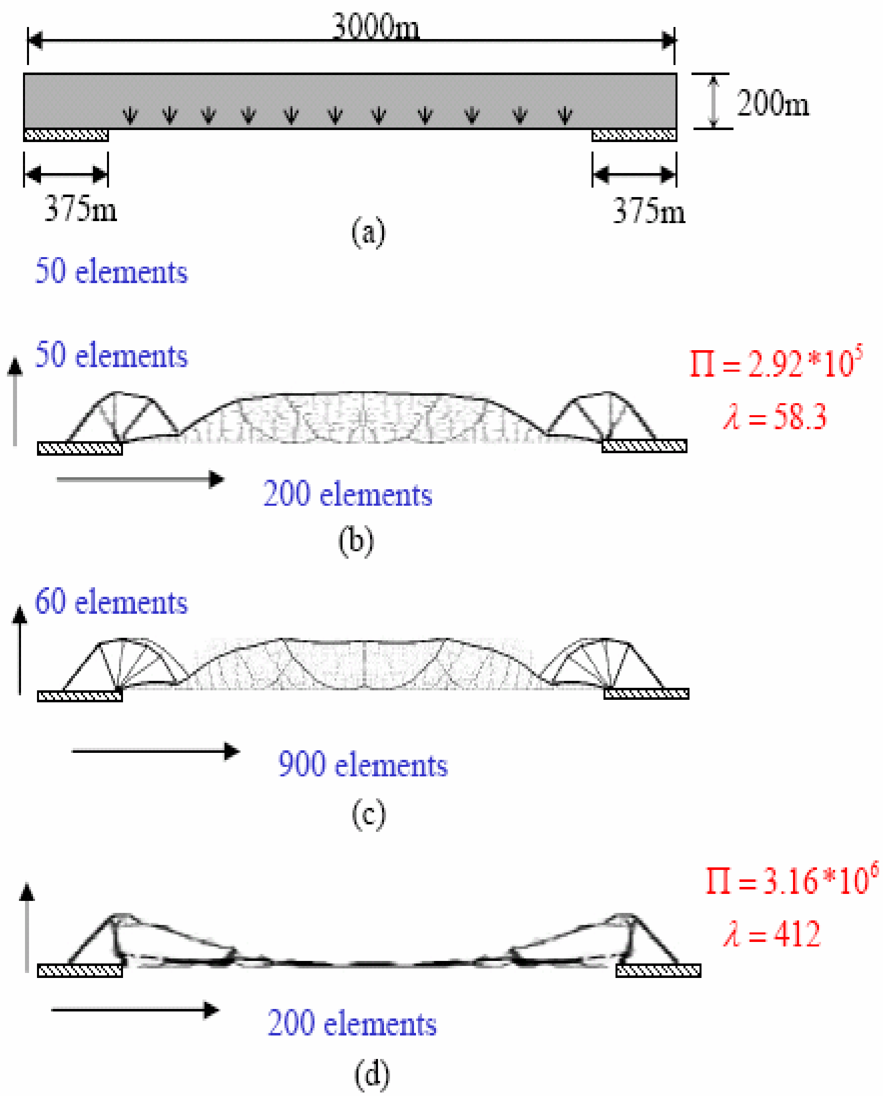
Long-Span Bridge Problem "2-Supports"

Design domain with 12.5% material constraint and 10 kPa applied load.

Compliance Minimization

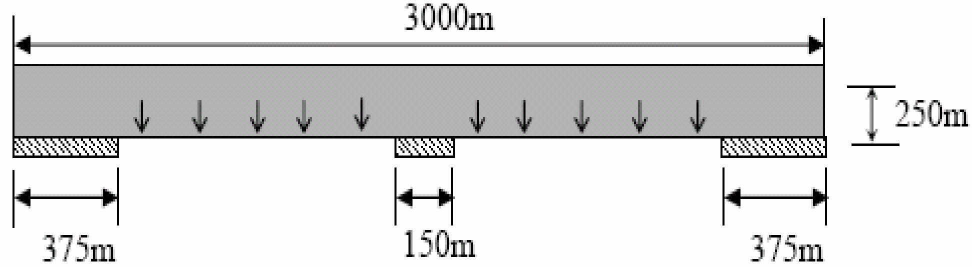
Compliance Minimization
(finer mesh)

Optimimum linearized buckling stability

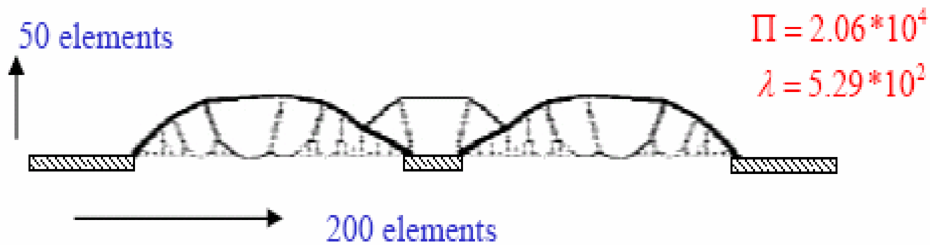


Long-Span Bridge Problem "3-Supports"

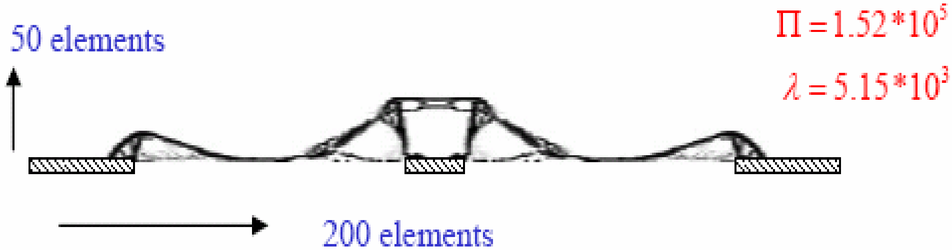
Design domain with 12.5% material constraint and 10 kPa applied load.



Compliance Minimization



Optimimum linearized buckling stability



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Current Research Issues:

- Designing Structures in 3-Dimensions
  - Iterative Eigensolvers
    - How to reliably solve: $K\phi+\lambda G\phi=0$ for the minimum eigenpair without ever forming or factorizing K or G?
- Design of Compliant Mechanisms
  - Micro-devices