Form-Finding of Long Span Bridges with Continuum Topology Optimization and a Buckling Criterion

Salam Rahmatalla (Graduate Student)
Prof. Colby C. Swan (Adviser)

Department of Civil and Environmental Engineering
Center for Computer-Aided Design
The University of Iowa

2003 STRUCTURES CONGRESS & EXPOSITION

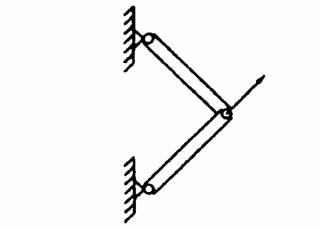
Objective Statement

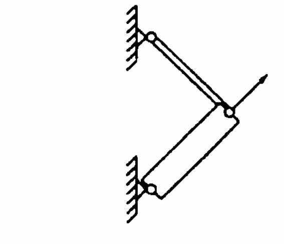
To develop a continuum topology formulation capable of finding structural forms of maximum buckling stability.

Introduction

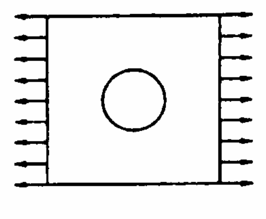
Structural Optimization

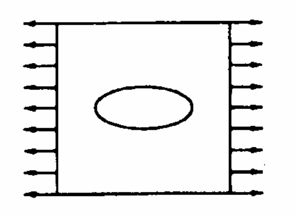
. Size Optimization



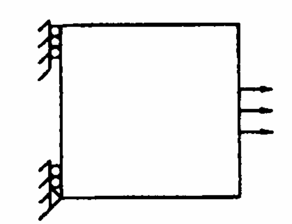


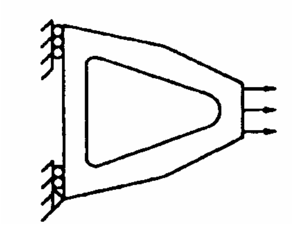
. Shape Optimization



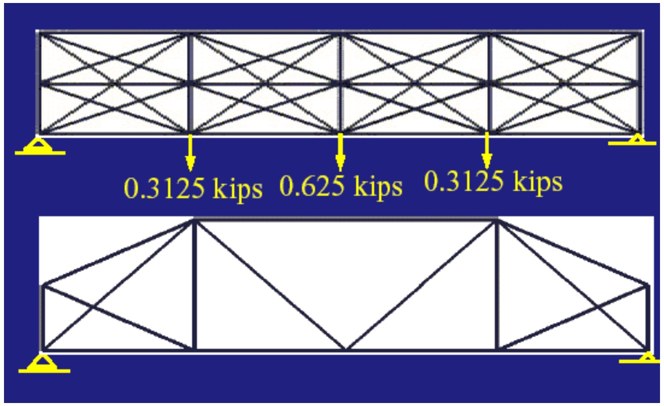


. Topology Optimization

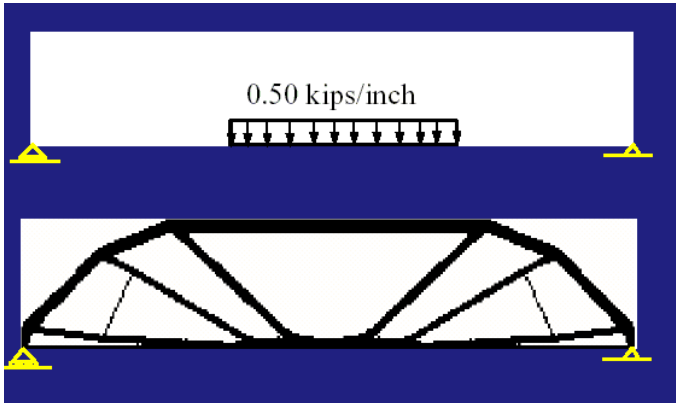




Alternative Topology Optimization Formulations

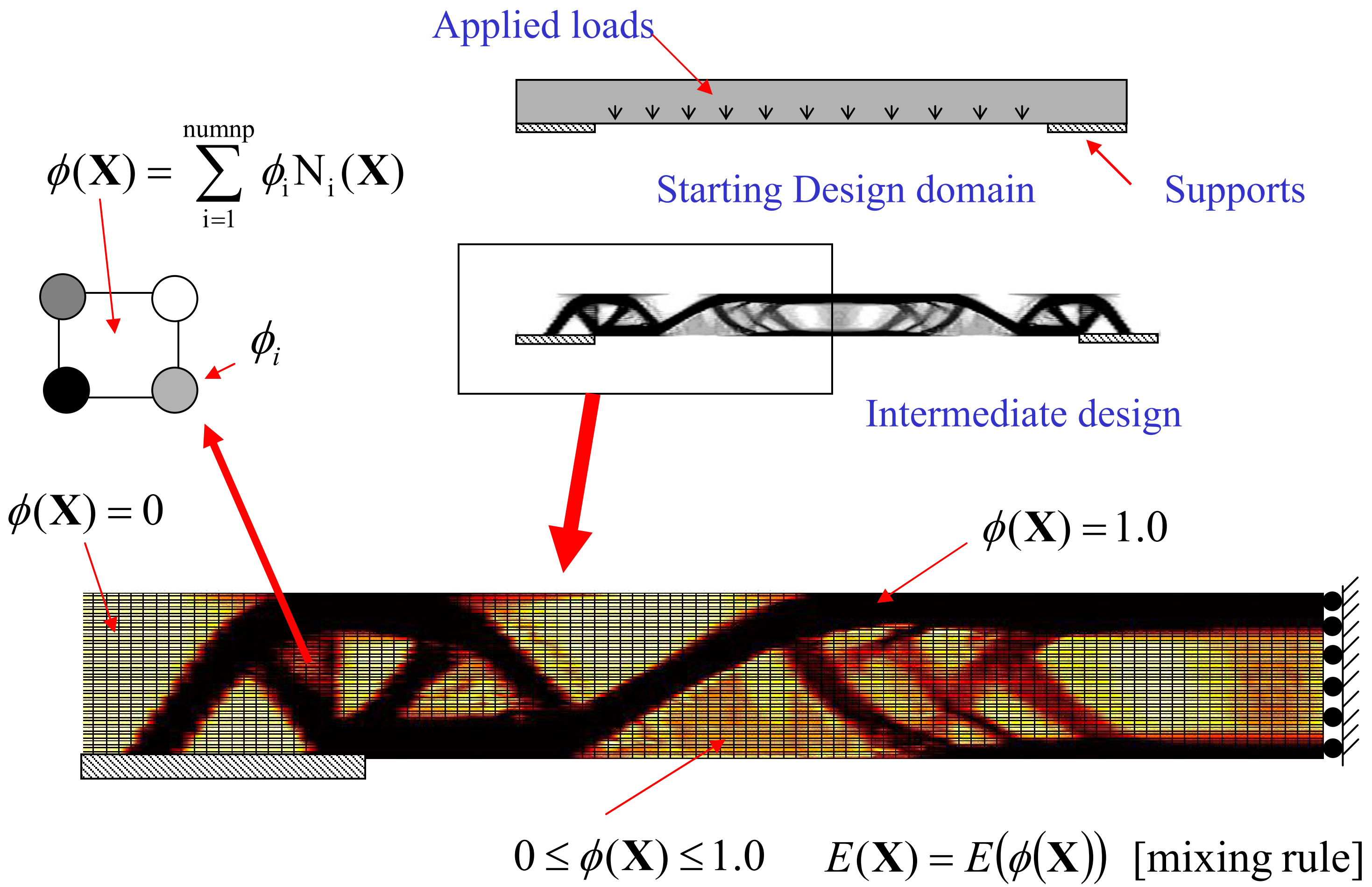


Discrete topology optimization
(Ground structures)



Continuum topology optimization

Elements of Continuum topology optimization



Sparsity of Long-Span Bridges

Sunshine Skyway bridge cable-stayed bridge in Tampa, Florida

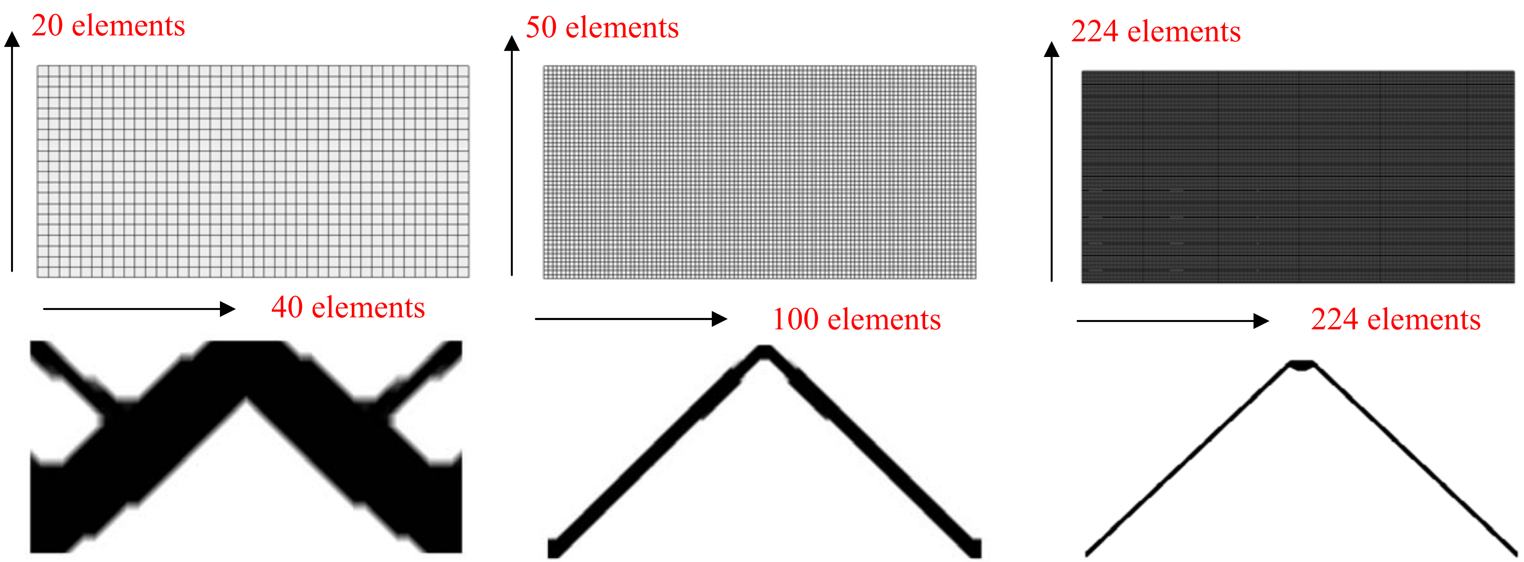
Akashi Bridge suspension bridge In Japan

Most long-span bridges occupy < 1% of their envelope volume.

SPARSITY in Topology Optimization

- Fixed-mesh model of full envelope volume;
  - must capture the form of the structure with realistic sparsity
  - must capture mechanical performance of the structure

- Fine meshes are required;
- Implies large computational expense;



50% Material constraint usage **10% Material constrain usage** **3% Material constraint usage**

2003 STRUCTURES CONGRESS & EXPOSITION

The University of Iowa

- Structure modeled as linearly elastic system
  - Stability analysis performed via linearized buckling analysis

- Linear elastic problem: $\mathbf{K}_L \cdot \mathbf{u} = \mathbf{f}^{ext}$
- Eigenvalue problem: $\boldsymbol{K}_L(\boldsymbol{b})\boldsymbol{\psi} + \lambda \boldsymbol{G}(\boldsymbol{u},\boldsymbol{b})\boldsymbol{\psi} = \boldsymbol{0}$ $\quad \lambda = -\dfrac{\boldsymbol{\psi}^T \boldsymbol{K}_L \boldsymbol{\psi}}{\boldsymbol{\psi}^T \boldsymbol{G} \boldsymbol{\psi}}$
- Objective function: $L(\boldsymbol{u},\boldsymbol{b}) = \dfrac{1}{\min(\lambda)}$
- Design sensitivity analysis:

$$\frac{dL}{d\boldsymbol{b}} = -\boldsymbol{\psi}^T\left(\frac{\partial \boldsymbol{G}}{\partial \boldsymbol{b}} + \frac{1}{\lambda}\frac{\partial \boldsymbol{K}_L}{\partial \boldsymbol{b}}\right)\boldsymbol{\psi} + (\boldsymbol{u}^a)^T\left(\frac{\partial \boldsymbol{K}_L}{\partial \boldsymbol{b}}.\boldsymbol{u} - \frac{\partial \boldsymbol{f}^{ext}}{\partial \boldsymbol{b}}\right)$$

$$\boldsymbol{K}_L \boldsymbol{u}^a = \boldsymbol{\psi}^T \frac{\partial \boldsymbol{G}}{\partial \boldsymbol{u}} \boldsymbol{\psi}$$

Design Constraints

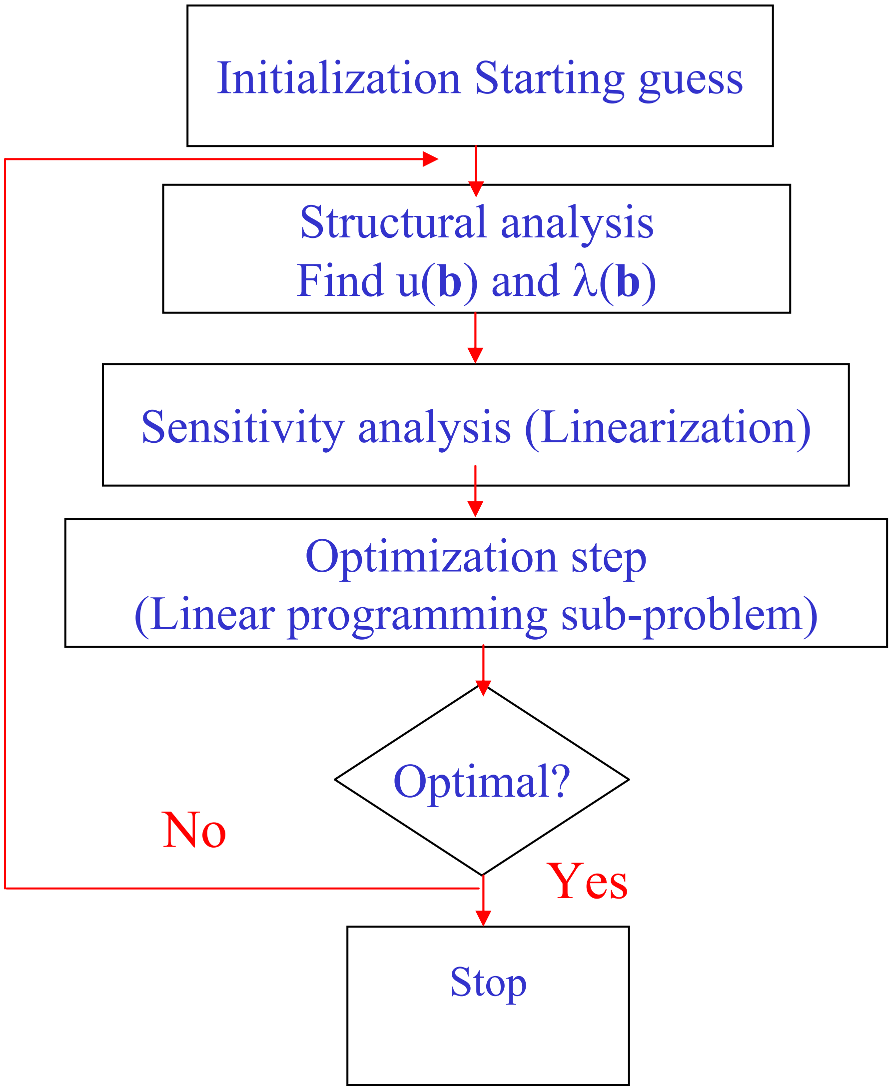
- Bounds on individual design variables

$$0 \le \phi_J \le 1 \quad J = 1,2,\ldots,NUMNP$$

- Material usage constraint

$$\frac{\int_{\Omega} \phi(\mathbf{X})\, d\Omega}{\int_{\Omega} d\Omega} \le C$$

Gradient-based Optimization Algorithm

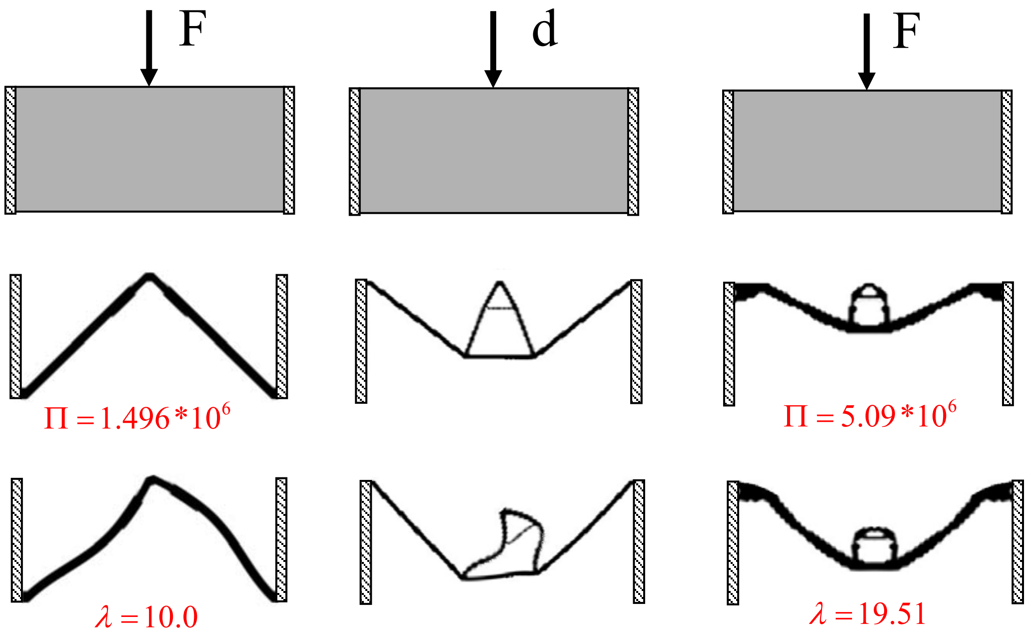


Fix-end Beam Problem

Design domain with 10% material constraint.

Resulting topology

First buckling mode.

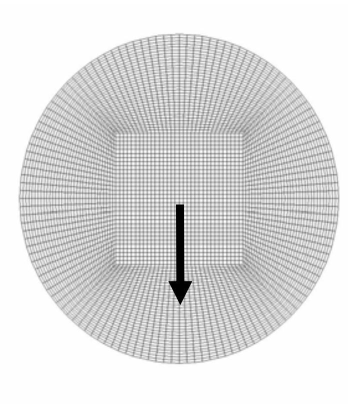


Minimizing the general compliance.

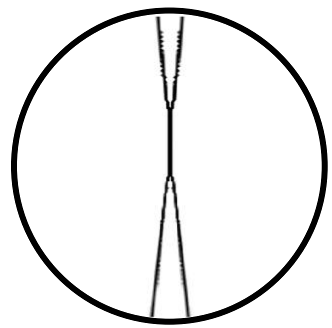
Maximizing the min. critical buckling load, nonlinear formulation.

Maximizing the min. critical buckling load, linearized buckling.

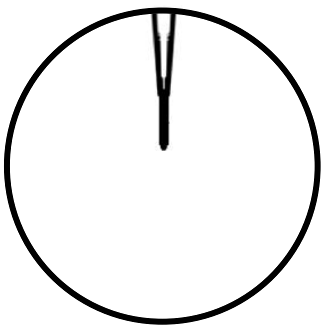
Circle Problem



Design domain
With material
2.5% Constraint

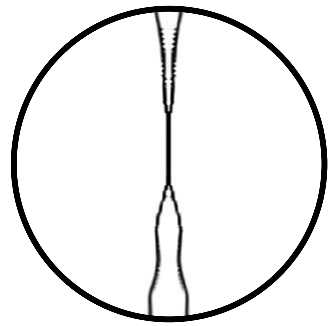


$\Pi = 4.09 * 10^5$



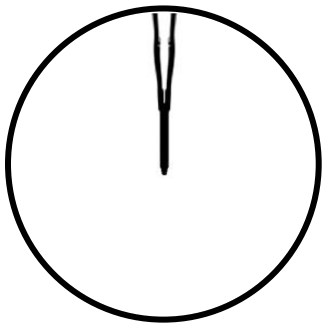
$\Pi = 2.66 * 10^5$

Undeformed configuration



$\lambda = 1.57 * 10^9$

Minimizing structural Compliance



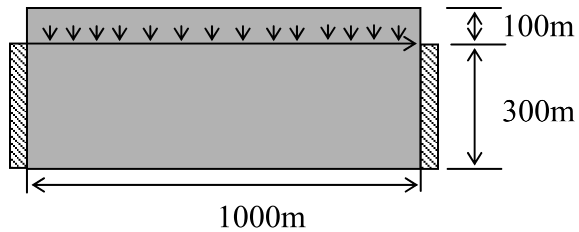
$\lambda = 3.28 * 10^9$

Maximizing the min. critical buckling load, linearized buckling.

Deformed configuration

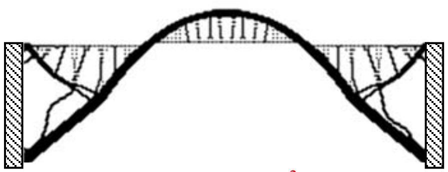
Canyon Bridge Problem

Design domain
With 12.5% material
Constraint and 10 kPa
applied load.



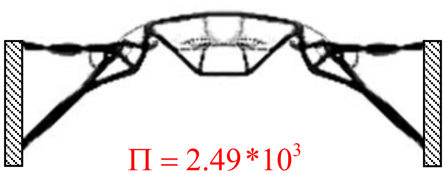
Minimizing the
general compliance

$\Pi = 1.7*10^3$

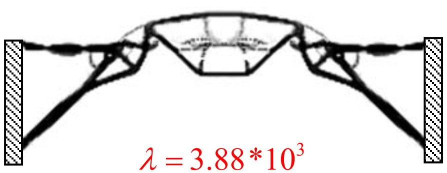


$\lambda = 4.84*10^3$

Maximizing the min.
critical buckling load,
linearized buckling.

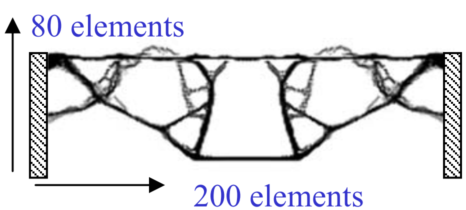


$\Pi = 2.49*10^3$

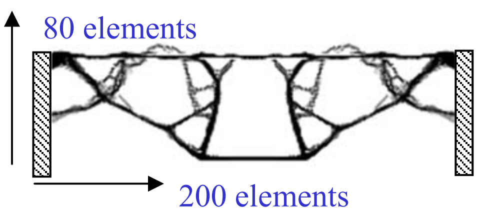


$\lambda = 3.88*10^3$

Maximizing the min.
critical buckling load,
linearized buckling.
(Non-designable deck)



$\Pi = 1*10^4$



$\lambda = 6.98*10^3$

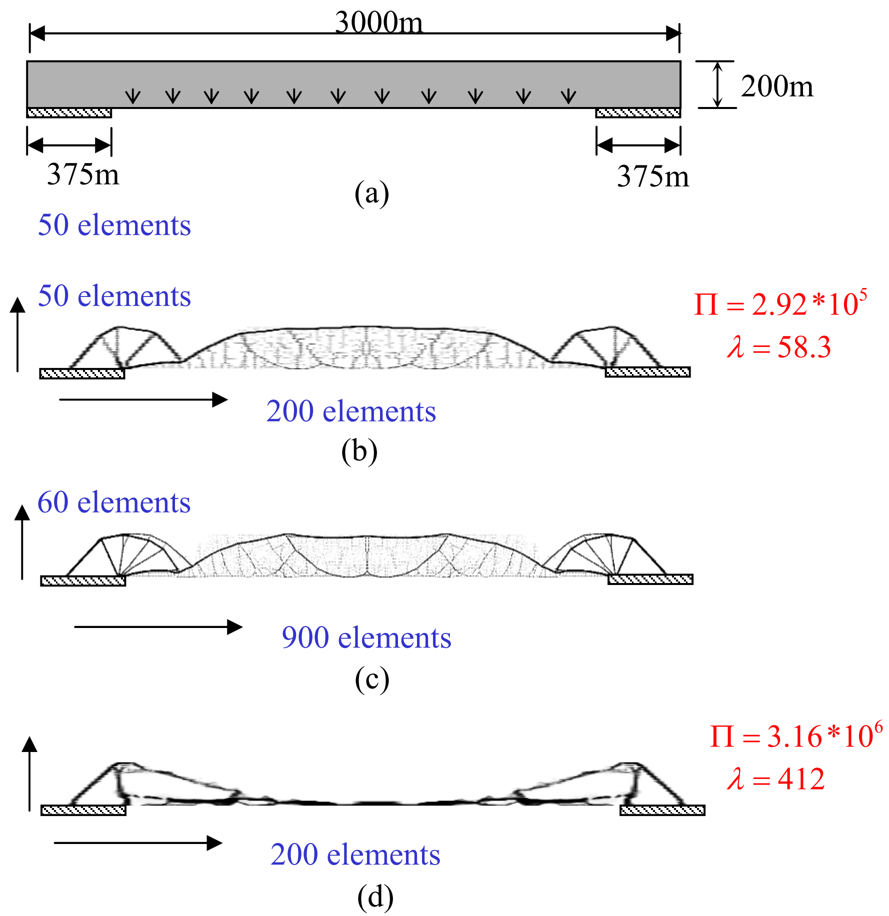
Long-Span Bridge Problem “2-Supports”

Design domain with 12.5% material constraint and 10 kPa applied load.

Compliance Minimization

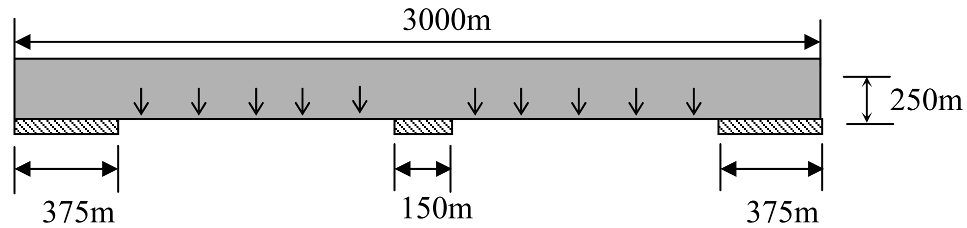
Compliance Minimization (finer mesh)

Optimimum linearized buckling stability

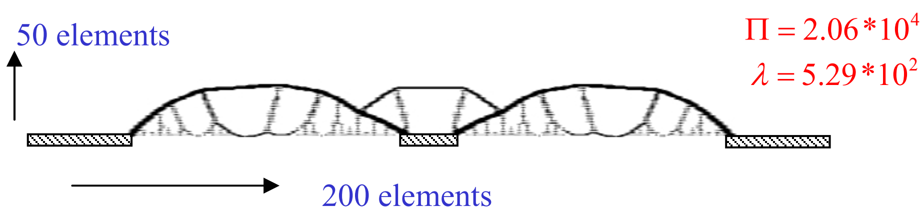


Long-Span Bridge Problem “3-Supports”

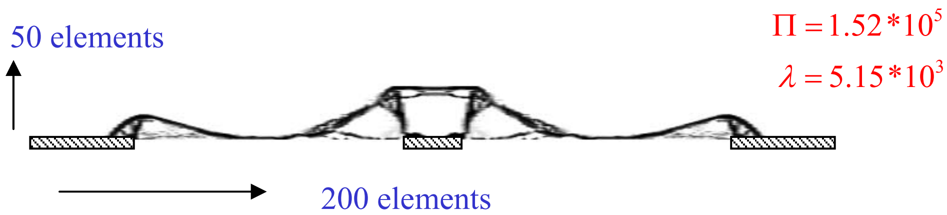
Design domain with 12.5% material constraint and 10 kPa applied load.



Compliance Minimization



Optimimum linearized buckling stability



Summary & Conclusion

- A formulation has been developed and tested for form-finding of large-scale sparse structures;
- The formulation is based on linearized buckling analysis;
- The structural form and topology are optimized to achieve maximum buckling stability;
- The formulation yields "concept designs" that resemble existing large-scale bridge structures;