Modeling Deformation-Induced Fluid Flow in the Lacunar-Canalicular System of Cortical Bone

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BACKGROUND

- Bones are adaptive living tissues:
  - Size & density increase and decrease with the intensity of mechanical loading
    - Intensified loading → bone growth
    - Reduced loading → bone atrophy

- Understanding the precise mechanisms in bone adaptation is an important objective in orthopedic research.
  - Potentially significant clinical implications

Hierarchical Structure of Whole Bone

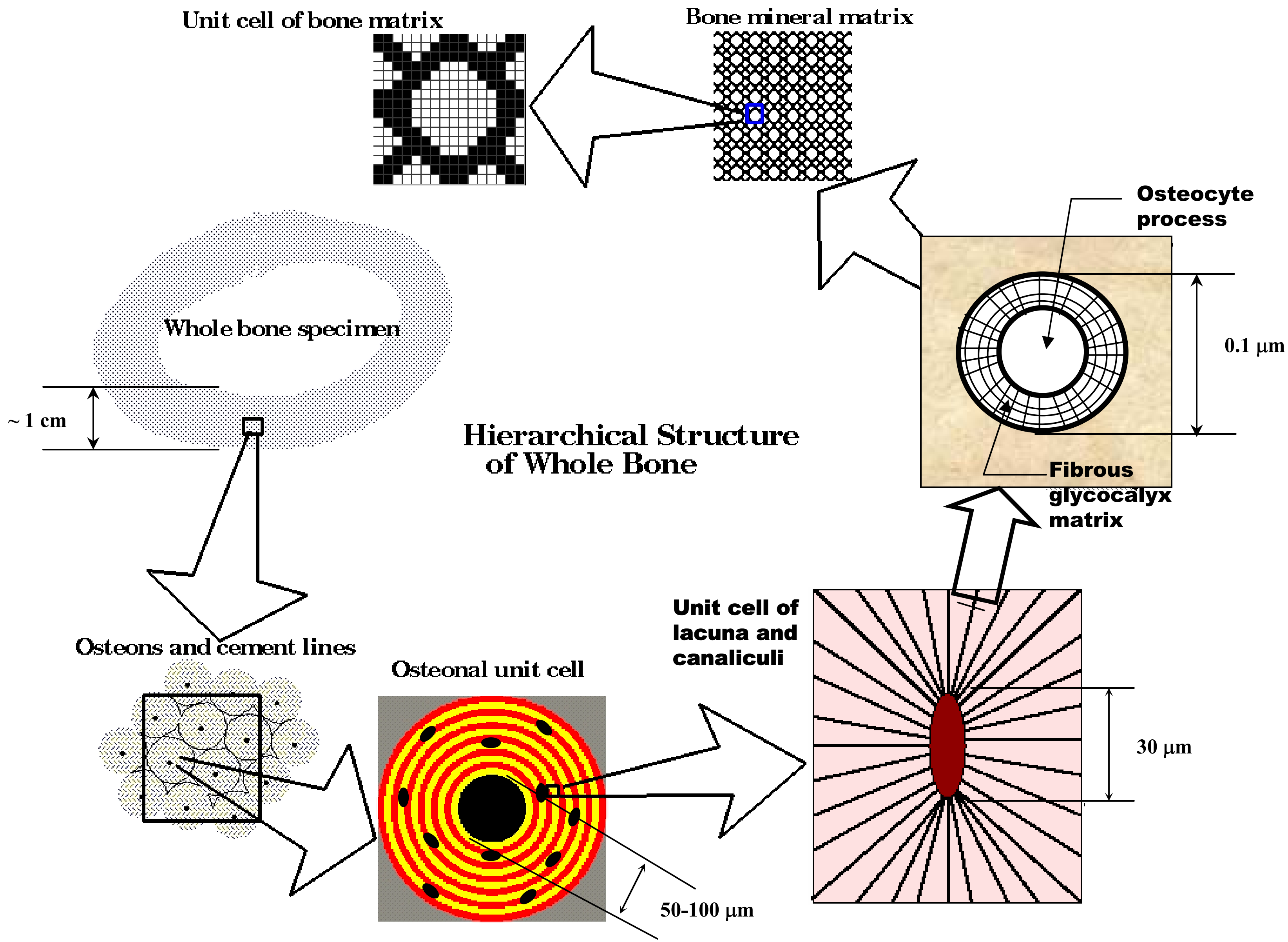
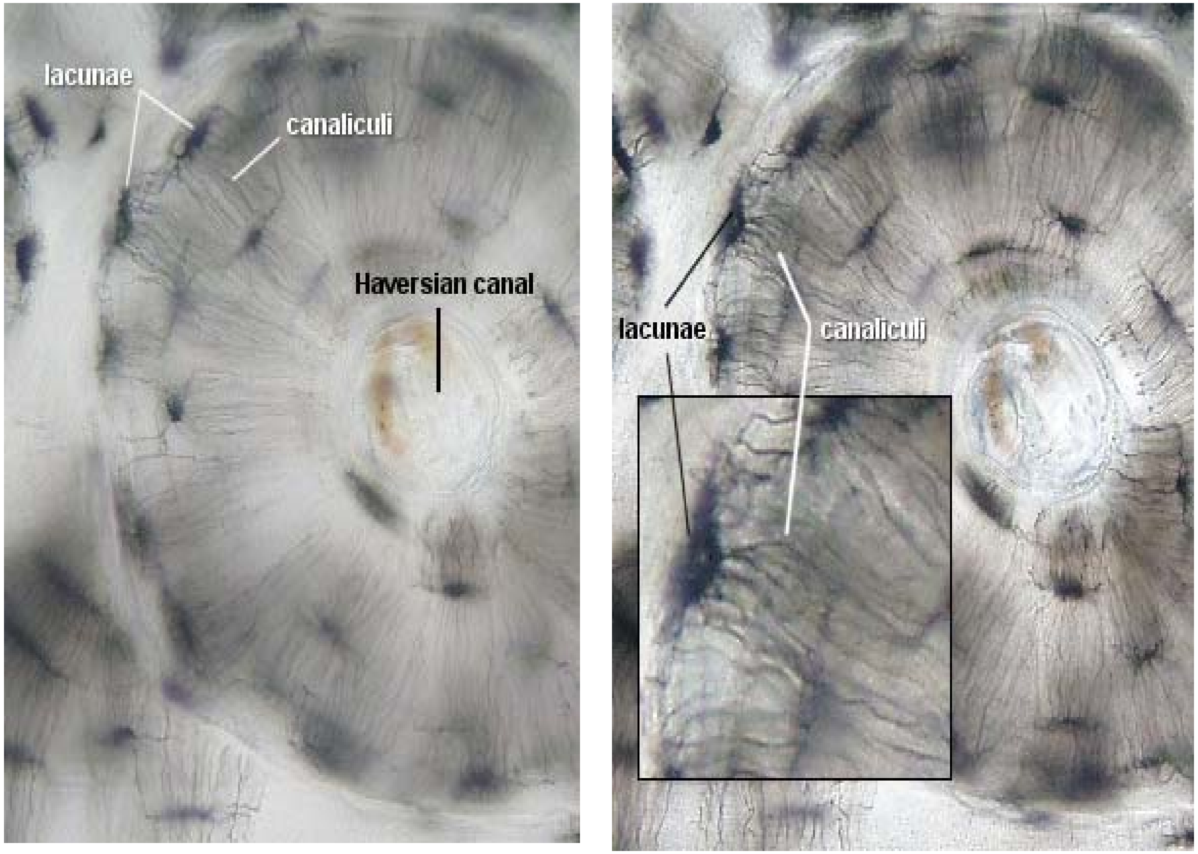


Photo-micrographs of osteon with Haversian canal, lacunae and canaliculi.

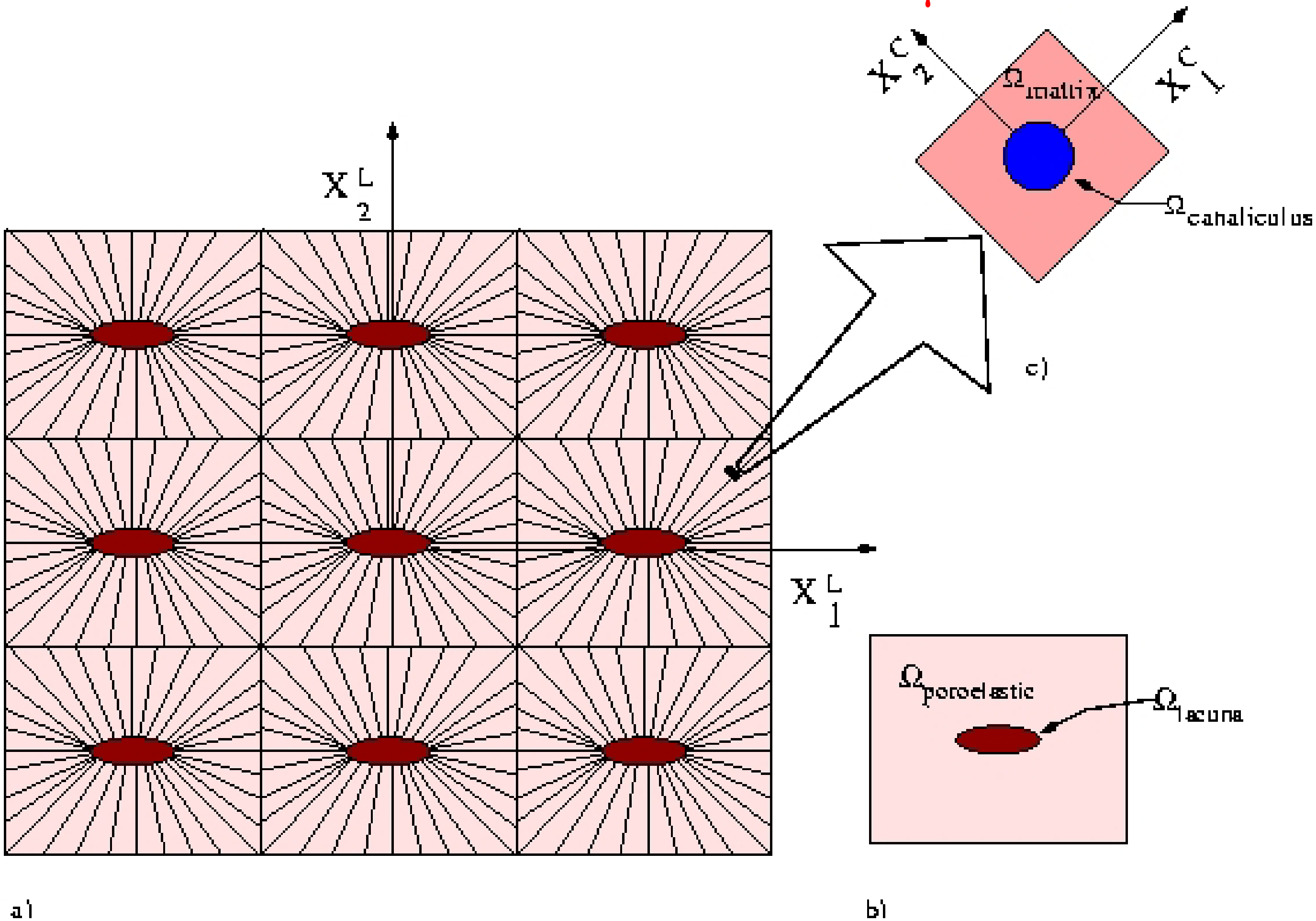


GOALS OF CURRENT RESEARCH

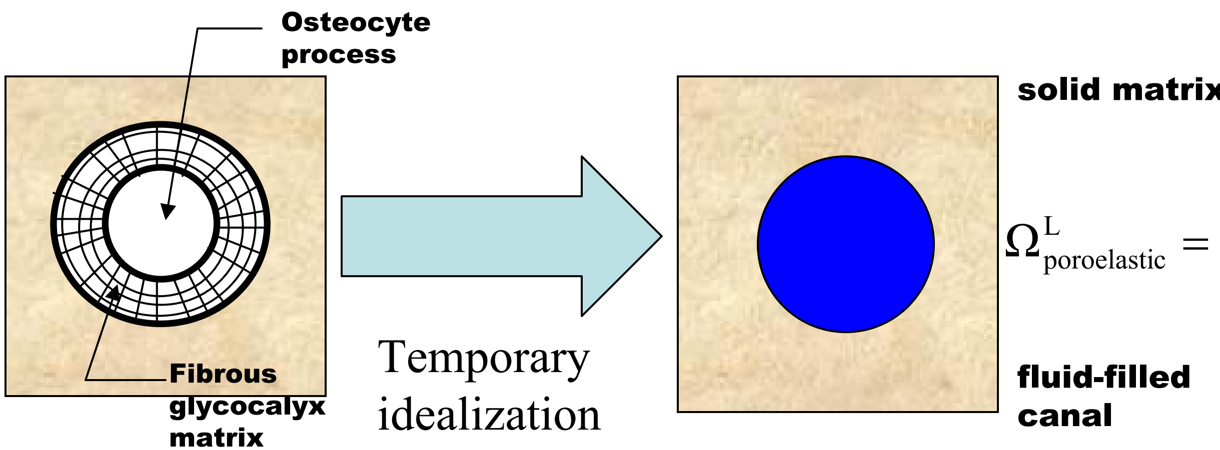
- Quantify nature of load-induced fluid flow in lacunar-canalicular system;
  - Magnitude of fluid pressures & shear stresses
- Compare pressures & stresses with those known to stimulate osteocyte response in *in vitro* cell culture studies.
- Adequately capture the heterogeneity and hierarchy of cortical bone.
  - address relevant multi-scale modeling challenges

Multi-Scale Modeling

- X^L denotes coordinate scale of lacunae ~ 50 μm
- X^C denotes coordinate scale of canaliculus ~ 0.5 μm



MICROMECHANICS OF CANALICULAR BONE



$$\Omega^{L}_{poroelastic} = \Omega^{C}_{matrix} \cup \Omega^{C}_{canaliculus}$$

$$\bar{\mathbf{v}}^{s}(\mathbf{X}^{L}) = \frac{\int_{\Omega^{C}_{matrix}} \mathbf{v}^{s} d\Omega}{V_{matrix}}; \quad \bar{\mathbf{v}}^{f}(\mathbf{X}^{L}) = \frac{\int_{\Omega^{C}_{canaliculus}} \mathbf{v}^{f} \, d\Omega}{V_{canaliculus}};$$

$$\bar{\mathbf{v}}(\mathbf{X}^{L}) = \phi^{C}_{matrix} \bar{\mathbf{v}}^{s} + \phi^{C}_{fluid} \bar{\mathbf{v}}^{f}$$

$$\bar{\mathbf{v}}(\mathbf{X}^{L}) = \bar{\mathbf{v}}^{s} + \dot{\bar{\mathbf{w}}}$$

$$\dot{\bar{\mathbf{w}}}\left(\mathbf{X}^{L}\right) = \varphi^{C}_{fluid} \left(\bar{\mathbf{v}}^{f} - \bar{\mathbf{v}}^{s}\right)$$

Averaged Strain of the Medium

$$\dot{\overline{\boldsymbol{\varepsilon}}}(\mathbf{X}^L) = \frac{1}{V}\left[\int_{\Omega_{matrix}} \dot{\boldsymbol{\varepsilon}}^{s}\left(\mathbf{X}^C\right) d\Omega_{\mathrm{matrix}} + \int_{\Gamma_{\mathrm{c-m}}} \tfrac{1}{2}\left[\mathbf{n}\otimes\mathbf{v}^{s} + \mathbf{v}^{s}\otimes\mathbf{n}\right] d\Gamma_{\mathrm{c-m}}\right]$$

$$= \varphi^{C}_{\mathrm{matrix}}\,\dot{\overline{\boldsymbol{\varepsilon}}}^{s} + \varphi^{C}_{\mathrm{fluid}}\,\dot{\overline{\boldsymbol{\varepsilon}}}^{\mathrm{canaliculus}}$$

$$= \tfrac{1}{2}\left(\nabla_{\mathbf{X}^L}\overline{\mathbf{v}} + \overline{\mathbf{v}}\nabla_{\mathbf{X}^L}\right)$$

Averaged Change in Fluid Content of the Medium

$$\dot{\zeta} = \frac{-1}{V}\int_{\Gamma_{\mathrm{canaliculus}}} \mathbf{n}\cdot\left[\mathbf{v}^{f} - \mathbf{v}^{s}\right] d\Gamma_{\mathrm{canaliculus}}$$

$$= -\nabla_{\mathbf{X}^L}\cdot\dot{\overline{\mathbf{w}}} = -\frac{\partial \dot{\overline{w}}_i}{\partial X^L_i}$$

Averaged Stresses in the Medium

$$\overline{\boldsymbol{\sigma}}^{s} = \frac{\int_{\Omega_{\mathrm{matrix}}} \boldsymbol{\sigma}^{s}\left(\mathbf{X}^C\right) d\Omega_{\mathrm{matrix}}}{V_{\mathrm{matrix}}}; \quad \overline{\boldsymbol{\sigma}}^{f} = \frac{\int_{\Omega_{\mathrm{canaliculus}}} \boldsymbol{\sigma}^{f}\left(\mathbf{X}^C\right) d\Omega_{\mathrm{canaliculus}}}{V_{\mathrm{canaliculus}}}; \quad \overline{p}^{f} = -\tfrac{1}{3}\,\mathrm{tr}(\overline{\boldsymbol{\sigma}}^{f})$$

$$\overline{\boldsymbol{\sigma}} = \varphi^{C}_{\mathrm{solid}}\overline{\boldsymbol{\sigma}}^{s} + \varphi^{C}_{\mathrm{fluid}}\overline{\boldsymbol{\sigma}}^{f}$$

Linear Poroelasticity Model (After Biot 1941, 1962)

$$\begin{bmatrix} \overline{\boldsymbol{\sigma}} \\ \overline{p}^{f} \end{bmatrix} = \begin{bmatrix} \mathbf{C} & \mathbf{G} \\ \mathbf{G}^{T} & Z \end{bmatrix} \cdot \begin{bmatrix} \overline{\boldsymbol{\varepsilon}} \\ \zeta \end{bmatrix}$$

- **C is the 6x6 symmetric undrained elastic matrix**
- **G is the 6x1 strain-pore-pressure coupling matrix**
- **Z is the scalar storage modulus**

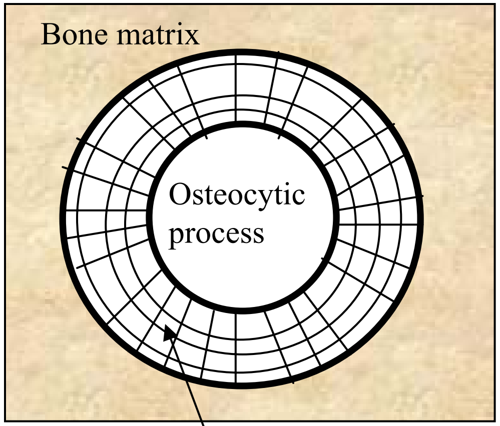
- **Procedure for computing C, G, Z:**
  - **Impose** $\overline{\boldsymbol{\varepsilon}}$ **and** ζ **on model;**
  - **Compute** $\overline{\boldsymbol{\sigma}}$ **and** $\overline{p}^{f}$ **;**

Effective poroelastic properties of canalicular bone matrix

X_3 direction is taken as aligned with the canaliculus.

$E_{bonematrix} = 11GPa$; $\nu_{bonematrix} = 0.38$; $\varphi_{canaliculus} = 0.02$; $\mathbf{D}_{canaliculi} = 0.10\mu m$		
Undrained Moduli (Gpa)		
$C_{11} = C_{22} = 19.46$; $C_{33} = 19.87$;	$C_{44} = C_{55} = 3.831$; $C_{66} = 3.811$;	$C_{12} = C_{21} = 11.85$; $C_{23} = C_{32} = 11.93$; $C_{13} = C_{31} = 11.93$;
Pore-pressure Coupling Moduli (Gpa)		
$G_1 = G_2 = -6.515$;	$G_3 = -5.280$;	
Storage Modulus (Gpa)		
$Z = 68.94$;		

Hydraulic Conductivity of Canalicular System



Assumption 1: Canaliculus fully occupied by fluid

$$k_{xx} = \frac{\varphi_{fluid}}{\rho^f \omega}\left[\frac{2}{\kappa}\frac{I_1(i^{1/2}\kappa)}{I_o(i^{1/2}\kappa)} - i\right] \xrightarrow[\kappa \ll 1]{} k_{xx} = \phi_{fluid} R^2 / 8\mu$$

where $\kappa = R/\beta = R\left[\mu/(\rho^f \omega)\right]^{-1/2}$

Assumption 2: Canaliculus occupied by osteocytic process and glycocalyx matrix. (Tsay and Weinbaum, 1991)

$$k = \left[\left(\frac{1}{3}\right)\frac{1}{k_{pl}} + \left(\frac{2}{3}\right)\frac{1}{k_{pz}}\right]^{-1} \quad k_{pl} = 0.147 a_0^2 \left(\frac{\Delta}{a_0}\right)^{2.285}; \quad k_{pz} = 0.0572 a_0^2 \left(\frac{\Delta}{a_0}\right)^{2.377};$$

a_o = fiber diameter (~ .6 nm)

Δ = fiber spacing (~ 7 nm)

(Weinbaum et al, 1994)

HYDRAULIC CONDUCTIVITIES USED IN COMPUTATIONS

	Axial permeability (m^2)	Transverse permeability (m^2)
Clear canaliculus assumption	$6.7 \cdot 10^{-18}$ m^2	$6.7 \cdot 10^{-21}$ m^2
Glycocalyx matrix assumption	$7.5 \cdot 10^{-20}$ m^2	$6.7 \cdot 10^{-21}$ m^2

Poroelastic Modeling at Lacunar Scale

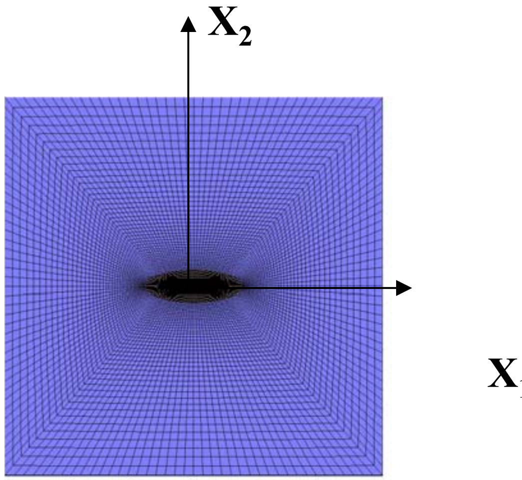
$$\bar{\sigma}_{ij,j} + \bar{\rho} b_j - \bar{\rho}\ddot{\bar{u}}_j - \bar{\rho}^f \ddot{\bar{w}}_j = 0$$

Total Medium Equation of Motion

$$-\bar{p}^f{}_{,j} + \bar{\rho}^f b_j - R_{ji}\dot{\bar{w}}_i - \bar{\rho}^f \ddot{\bar{u}}_j - \frac{\bar{\rho}^f}{\varphi^C_{fluid}}\ddot{\bar{w}}_j = 0$$

Fluid Equation of Motion

Strained-Controlled Poroelastic Unit Cell Analysis



$$\bar{\boldsymbol{\sigma}}(\mathbf{Y}) = \bar{\bar{\boldsymbol{\sigma}}} + \bar{\boldsymbol{\sigma}}^*(\mathbf{Y}); \qquad \bar{\boldsymbol{\varepsilon}}(\mathbf{Y}) = \bar{\bar{\boldsymbol{\varepsilon}}} + \bar{\boldsymbol{\varepsilon}}^*(\mathbf{Y})$$

$$\bar{p}^f(\mathbf{Y}) = \bar{\bar{p}}^f + \bar{p}^{f*}(\mathbf{Y}); \qquad \zeta(\mathbf{Y}) = \bar{\zeta} + \zeta^*(\mathbf{Y})$$

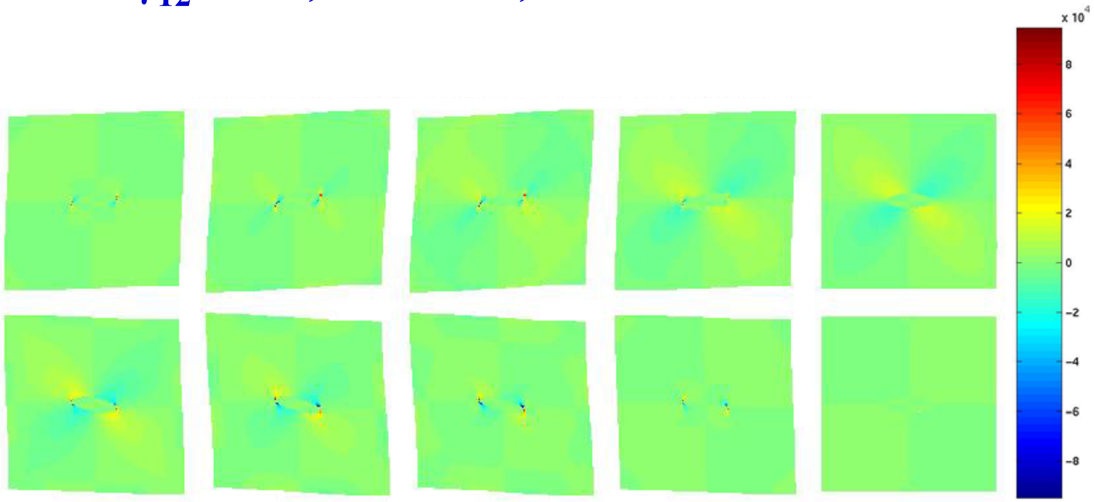
Method:

- **Impose history of $\bar{\bar{\boldsymbol{\varepsilon}}}$ and $\bar{\zeta}$ on unit cell model;**
- **Compute corresponding histories of $\bar{\boldsymbol{\varepsilon}}, \zeta, \bar{\boldsymbol{\sigma}}, \bar{p}^f$.**

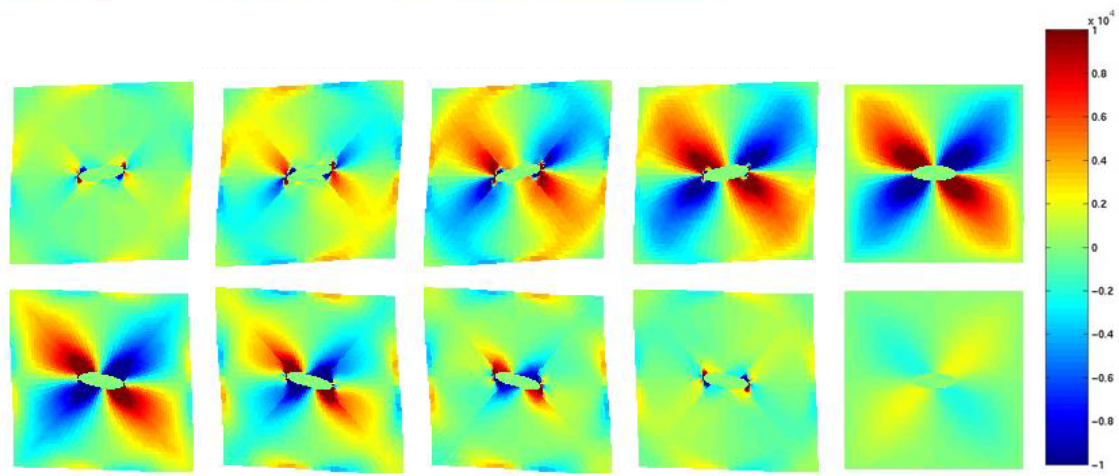
Fluid Pressure Distribution Under Harmonic Shear Loading of Single-Lacuna Unit Cell Model

- $\gamma_{12}=10^{-4}$; f = 1 Hz;

True fluid pressures



Truncated fluid pressure display ($p_f \leq 10$kPa)



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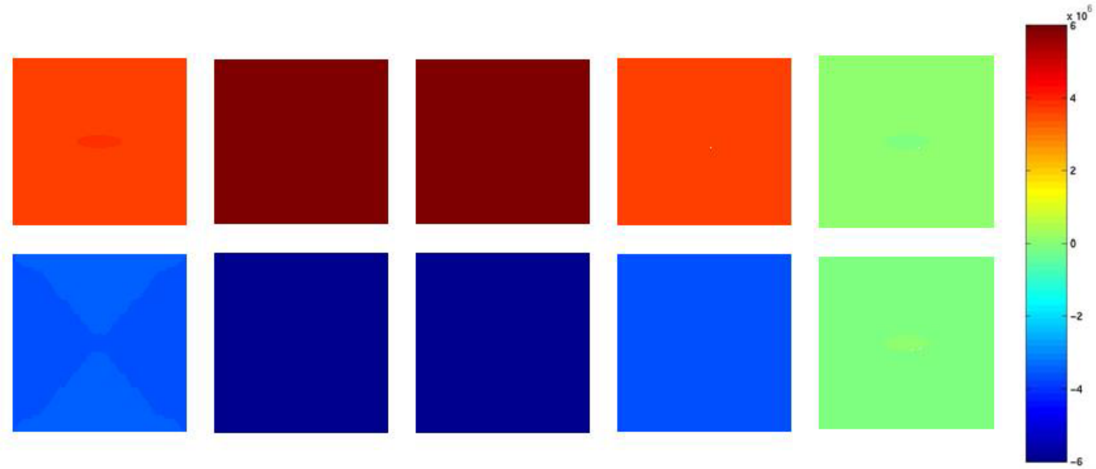
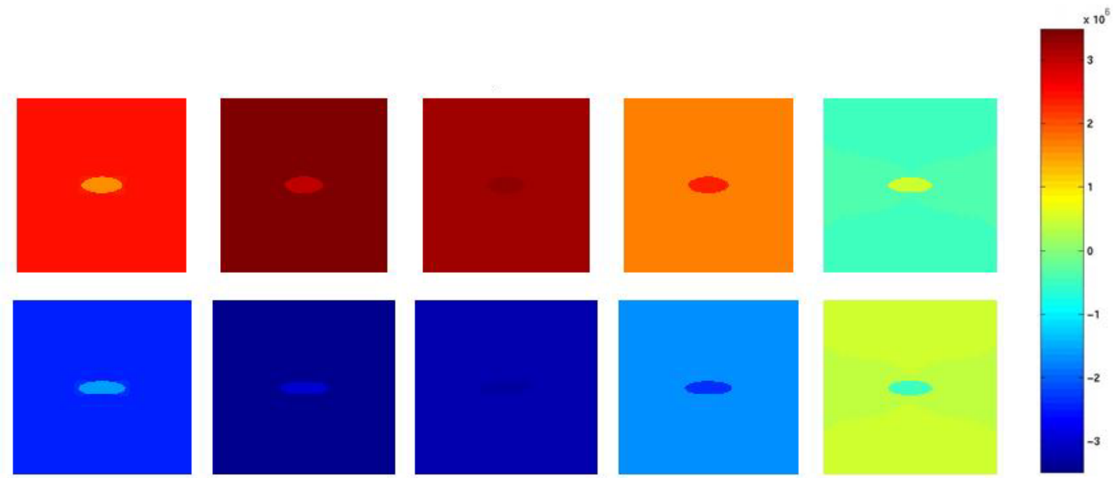
Fluid Pressure Distributions Under Harmonic Extensional Loadings of Single-Lacuna Unit Cell Model at f = 1 Hz;

$\varepsilon_{22}=10^{-4}$

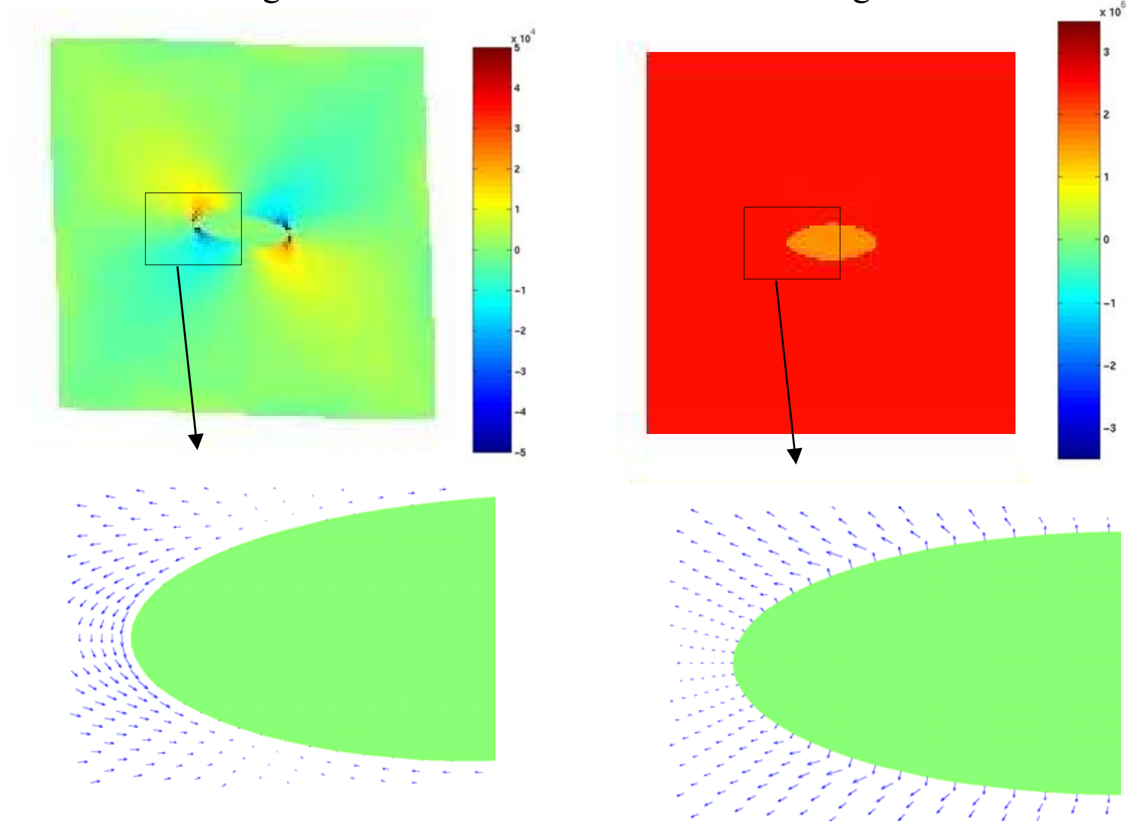


$\varepsilon_{11}=10^{-4}$



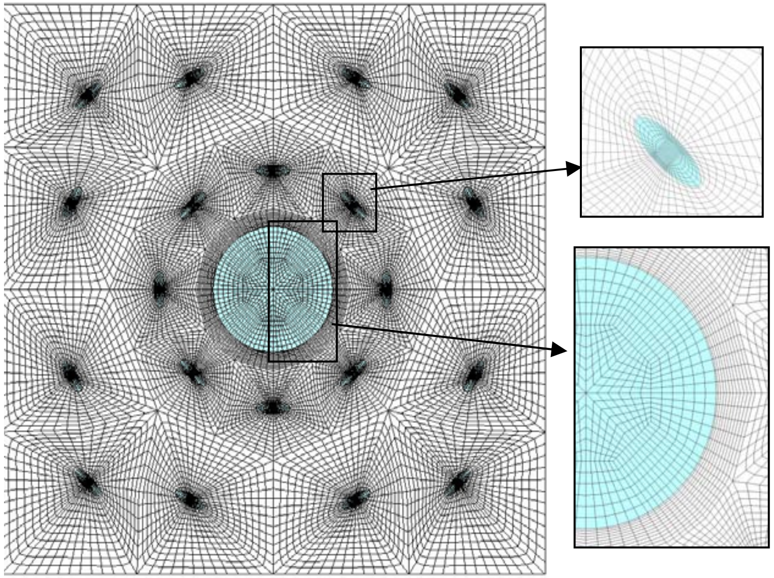
Local Fluid Flows in Vicinity of Lacuna:

Shear loading | Axial loading

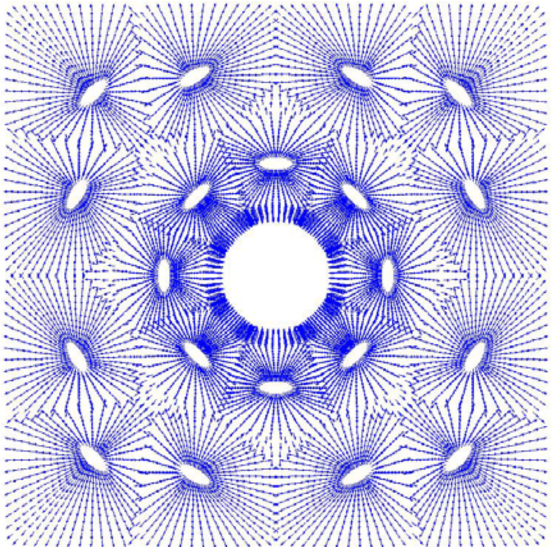


OSTEONAL MODEL

- 4% Haversian porosity
- 4% Lacunar porosity
- 2% Canalicular porosity



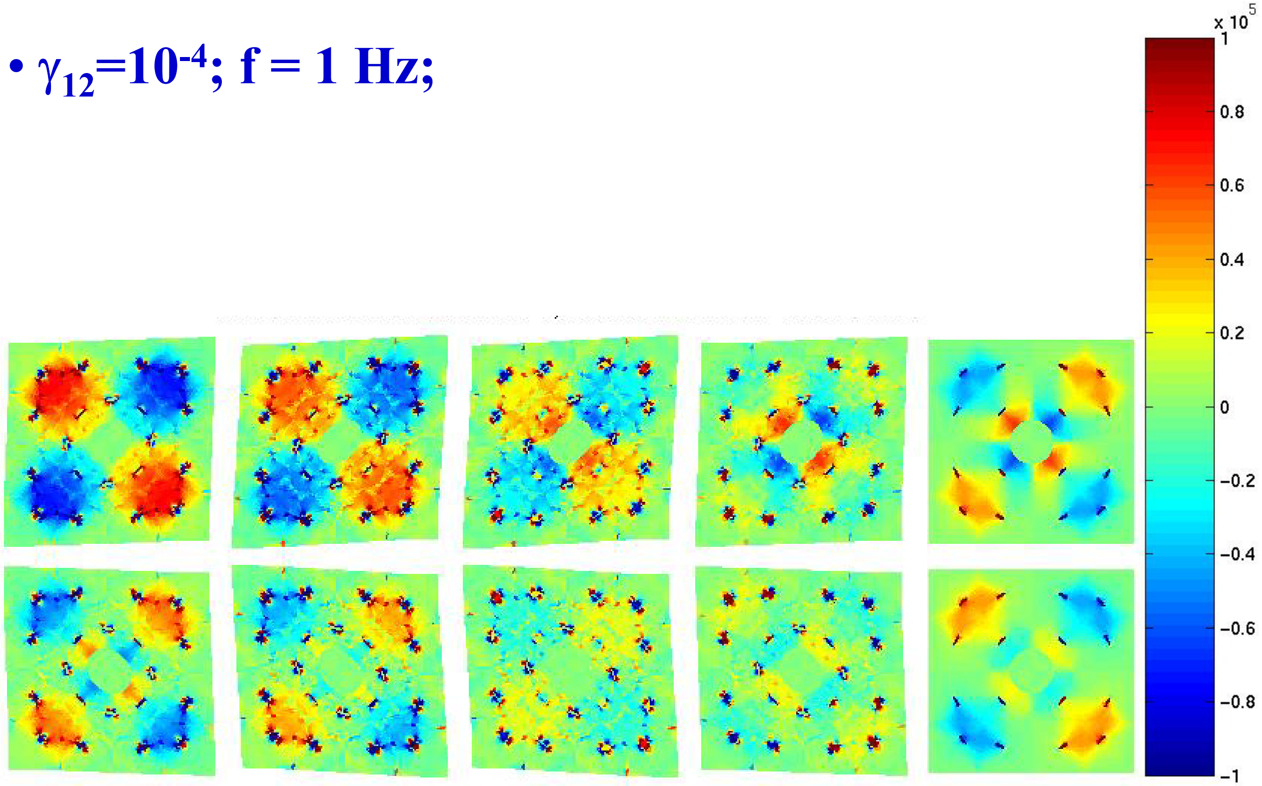
FEM Mesh of Model



Canalicular orientations in bone matrix

Fluid Pressure Distribution Under Harmonic Shear Loading of Osteonal Unit Cell Model

- $\gamma_{12}=10^{-4}$; f = 1 Hz;



Summary & Conclusions

- Significant fluid pressures and flow do occur in the lacunar-canalicular system:
  - At physiologically meaningful frequencies (10^0 Hz < f < 10^2 Hz);
  - At physiologically meaningful strains $\varepsilon \sim O(10^{-4})$;

- Remaining challenges (among many):
  - Begin to incorporate osteocyte (cell-mechanics) models;
  - Incorporate chemical transport phenomena;